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Fundamental properties of attack relations in structured argumentation with priorities ☆

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ABSTRACT

Due to a proliferation and diversity of approaches to structured argumentation with prioritized rules, several simple and intuitive principles for characterization and evaluation of the proposed attack relations have recently been introduced in [23]. While the proposed principles and properties are helpful, they do not identify unique attack relations. Any user of structured argumentation still faces a fundamental problem of determining an appropriate attack relation for her/his application and further principles that could help in identifying such attack relation.

We argue that a key purpose of introducing priorities between defeasible rules is to remove undesired attacks while keeping the set of removed attacks to a minimum. This intuitive idea could be viewed as a kind of minimal-removal-principle. We show in this paper that the minimal-removal-principle together with a new simple and intuitive property of inconsistency-resolving and previously proposed properties indeed characterize a unique attack relation referred to as the canonical attack relation. We show that canonical attack relations could be characterized in three distinct ways, as the supremum of a complete upper-semilattice of regular attack relations, or by removing the undesired attacks from the basic attack relations where the undesired attacks are captured by a least-fixed point of an intuitive removal function, or as the normal attack relations introduced in an earlier paper for a class of well-prioritized knowledge bases.

We start our study with a language consisting only of literals and two type of attacks, rebut and undercut. We then show that our approach can easily be scaled up by showing that all key results still hold for general underlying logical languages and the inclusion of assumptions.

We apply our proposed approach to valued-based argumentation and show that it also leads to the canonical semantics.

1. Introduction

At its most abstraction, an argument system could be viewed as an argumentation framework [29] consisting of a set of arguments and a binary attack relation between them. Though simple, argumentation frameworks are powerful enough to

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provide a sophisticated account of the acceptance of arguments representing different ways people could draw conclusions from exchanges of arguments.

While there is a good understanding about the acceptability of arguments due to an extensive amount of research [29, 7,6,8,34,35,14,2,19], more needs to be done to gain a better understanding about the structure of arguments and their attack relations. In domains like experimental medicine, arguments often have no internal structure as the purpose of the experiments is to uncover the underlining rules [41]. In contrast, arguments in commonsense reasoning and legal domains are often based on complex sets of rules [9,26]. The complex structure of arguments often leads to challenging questions about the structure of their attack relations.

There are extensive research on rule-based systems with prioritized rules (see for example [44,43,51,4,3,13,22,50,48,15, 39,38,49]). Amgoud&Cayrol [4] have proposed the use of priorities between rules to filter out certain undesired attacks. Continuing this idea, Prakken and Modgil&Prakken [47,43,44] have proposed ASPIC+, a rich framework for structured argumentation with prioritized rules with several distinct systems of attack relations. This rich diversity of proposed attack relations also poses a serious challenge for any potential user of structured argumentation as such a user would have to decide which attack relation should be selected and implemented for her/his domain.

For illustration, we recall and elaborate a simple example from [23] that is helpful in explaining many concepts and ideas later.

Example 1 *(A Sherlock Holmes investigation).* Sherlock Holmes is investigating a case involving three persons P_1, P_2 and S together with the dead body of a big man. The case could be represented by the following knowledge base.

1. The knowledge that one of the persons is the murderer is represented by three strict rules:[1]

$$r_1 : Inno(P_1), Inno(S) \to \neg Inno(P_2)$$

$$r_2 : Inno(P_2), Inno(S) \to \neg Inno(P_1)$$

$$r_3 : Inno(P_1), Inno(P_2) \to \neg Inno(S)$$

2. S is a small child who cannot kill a big man. This fact is captured in the base of evidence $BE = \{Inno(S)\}$.
3. The legal principle that people are considered innocent until proven otherwise could be represented by three defeasible rules

$$d_1 :\Rightarrow Inno(P_1) \qquad d_2 :\Rightarrow Inno(P_2) \qquad d :\Rightarrow Inno(S)$$

4. After digging around, it becomes clear to Holmes that P_1 has a strong motive to kill the victim while there is nothing connecting P_2 to the dead man. He hence will focus his investigation on P_1. This knowledge is represented by a preference

$$d_1 \prec d_2$$

stating that Holmes gives higher priority (in his investigation) to the scenario in which P_2 is innocent than to the other one.

Let KB be the knowledge base containing the strict rules r_1, r_2, r_3, the three defaults d_1, d_2, d and the fact that S is innocent together with the preference $d_1 \prec d_2$.

Relevant arguments concerning the innocence of P_1, P_2 wrt KB are given in Fig. 1.

Due to the preference of d_2 over d_1, N_2 attacks A_1 wrt all four MP-attack relations[2] in the ASPIC+ framework. Therefore N_2 also attacks N_1, N'_1. Hence there is a stable extension containing A_2, N_2, N'_2 implying the expected conclusion $\neg Inno(P_1), Inno(P_2)$.

According to the MP-attack relations based on the democratic order, N'_1 attacks A_2.[3] Hence N'_1 also attacks N_2, N'_2. Therefore there is another stable extension containing A_1, N'_1, N_1 justifying $Inno(P_1), \neg Inno(P_2)$, a rather counter-intuitive set of beliefs.

Even though N'_1 attacks A_2 wrt MP-attack relations based on the democratic order, N_1 does not. This is rather surprising as N'_1 could be seen as a weakened version of N_1 where the undisputed fact "S is innocent" in N_1 is replaced by a defeasible one in N'_1 stating only that if there is no evidence to the contrary then S is innocent. As it has been pointed out in [23], MP-attack relations based on the democratic order do not satisfy the property of attack monotonicity stating that if a weakened version of an argument A attacks an argument B then A itself should also attack B. We can say the reason that

[1] Inno stands for Innocent.

[2] For ease of reference, we refer to the attack relations proposed and studied by Modgil and Prakken in [43] as MP-attack relations in the rest of this example. See section 8.1 for their precise definitions.

[3] To be precise, N'_1 attacks A_2 wrt the democratic order because the conclusion of N'_1 contradicts the conclusion of A_2 and N'_1 is not less preferred than A according to the democratic order (i.e. there is a defeasible rule (d) in N'_1 that is not less preferred than d_2). See section 8.1 for the precise definition.

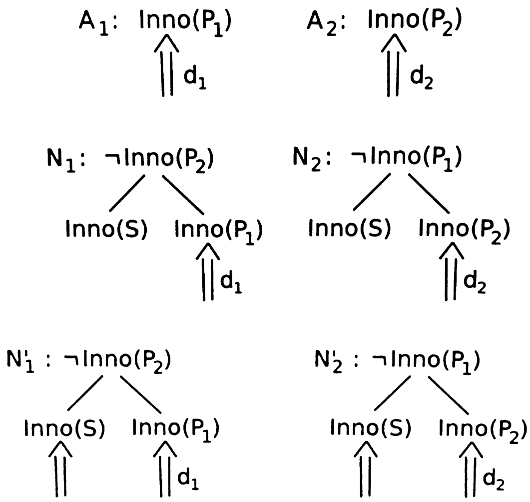

Fig. 1. Sherlock Holmes arguments.

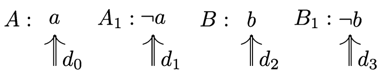

Fig. 2. Minimal removal.

MP-attack relations based on democratic order support counter-intuitive beliefs in this example is due to a violation of this property.

The example illustrates the need to establish general principles for the characterization and evaluation of alternative attack relations for rule-based systems.[4] Caminada and Amgoud [17] have introduced the postulates of consistency and closure for argument-based systems. A subargument closure postulate stating that any extension should contain all subarguments of its arguments has been studied by Martinez&Garcia&Simari [42], Amgoud [1], Modgi&Prakken [43]. Though the three proposed postulates are very helpful, they are not sufficient to guarantee intuitive semantics, as they do not take into account the preferences of defeasible rules. To address this problem, Dung [30,23] has proposed a set of simple and intuitive properties, referred to as ordinary properties in [23] and showed that they can be used to characterize and evaluate the proposed attack relations in structured argumentation with prioritized rules.

As there could be in general many attack relations satisfying the ordinary properties, there still remains a huge challenge for any user/developer of structured argumentation with prioritized rules to decide which of the ordinary attack relations should be selected/implemented for her/his domain and what are the guidelines for picking an appropriate one.

Amgoud&Cayrol [4] were arguably the first to study the application of priorities of defeasible rules to define a preference relation between arguments and then using the preference relation to define attack relation between arguments. Prakken [47] and Modgil&Prakken [43] distinguished between preference independent and preference dependent attacks and similar to Amgoud&Cayrol [4], applied priorities to define a preference relation between preference dependent arguments. In essence, we can say that both Amgoud&Cayrol [4] and Modgil&Prakken [43] have applied priorities between defeasible rules to filter out undesired attacks. In this paper, we argue further that the removal of attacks should be kept to a minimum. This intuitive idea could be viewed as a kind of minimal-removal-principle.

The following very simple example illustrates the idea.

Example 2. Consider a knowledge base consisting of just four defeasible rules and four arguments A, A_1, B, B_1 as seen in Fig. 2. Without any preference between the rules, we have A, A_1 attack each other. Similarly B, B_1 attack each other.

Suppose that for whatever reason d_3 is strictly less preferred than d_2 (i.e. $d_3 \prec d_2$). The introduction of the preference $d_3 \prec d_2$ in essence means that the attack of B_1 against B should be removed, but it does not say anything about the other attacks. Hence these other attacks should be kept, i.e. the attacks that should be removed should be kept to a minimum.

We introduce in this paper an intuitive property of inconsistency-resolving providing a deep structural insight into the nature of attack relations satisfying the postulate of consistency. We refer to the inconsistency-resolving property and the ordinary properties other than the credulous cumulativity as regular properties.

[4] An extensive discussion on this topic could be found in [23].

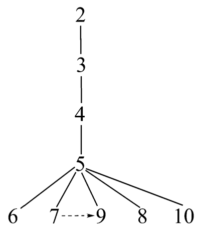

Fig. 3. Dependency graph.

It turns out that the minimal-removal principle coupled with regular properties indeed determine a unique attack relation that can be viewed as the common attack relation by any user/developer who agrees with the regular properties. Formally, we show that attack relations satisfying the regular properties form a complete upper-semilattice whose supremum referred to as the canonical attack relations could be viewed as uniquely identified by the minimal-removal principle. We also show that the canonical attack relations can be characterized by the least-fixed point of a removal-function defined by interpreting the regular properties contrapositively.

A class of normal attack relations has been introduced in [23]. Since every stable extension wrt any ordinary attack relation is also stable wrt a normal attack relation, normal attack relations are proposed as a kind of normal form for stable semantics, i.e. a kind of a standard candidate for selection as their attack relation by those users/developers of structured argumentation who adopt the stable extensions as their semantics. An interesting question is whether or under which conditions normal attack relations capture the intended attack relations of prioritized rules wrt to any extension semantics, not just stable semantics. More precisely, we are interested in the question of whether normal attack relations satisfy the property of consistency-resolving and the principle of minimal removal and whether or under which conditions normal attack relations capture the canonical attack relations. As determining whether an argument normal-attacks another is both simple and efficient, this question is hence both theoretically interesting and practically relevant.

When introducing preferences between defeasible rules, one key question is whether the introduced preferences are helpful and sensible. We introduce a class of well-prioritized systems and show that for this class, the canonical attack relation assignments and the normal attack relation assignment are identical. This result is important as determining whether an argument normal attacks another is very simple. Moreover we show that the canonical and normal attack relation assignments are equivalent wrt the stable semantics, i.e. they deliver the same sets of stable extensions.

In order to provide a simpler and more focused presentation of the essential notions of our proposal, we first introduce them in the context of a basic system that is expanded further later. In particular, we start with a language consisting only of literals and two type of attacks, rebut and undercut. We then show that our approach can easily be scaled up by showing that all key results still hold for general underlying logical languages and the inclusion of assumptions.

The paper is organized as follows. We recall in the next section the key concepts and notions of argumentation and defeasible knowledge bases on which the paper is based. We then introduce the important property of inconsistency-resolving and discuss sufficient conditions for the postulates of consistency and closure. In section 4, we introduce the concepts of regular attack relations and regular attack relation assignments. In section 5, we study the semilattice of regular attack relation assignments and propose the canonical attack relations. We formalize the minimal removal intuition by providing a least fixed-point characterization of the canonical attack relation assignments in section 6. We study a relevant class of well-prioritized rule-based systems for which canonical attack relations and normal attack relations coincide in section 7. We discuss the relationship of our approach to others in section 8. We show in section 9 that our approach can easily be scaled up for general underlying logical languages and the inclusion of assumptions. We conclude in section 10.[5] For ease of reading, the graph in Fig. 3 indicates the dependency between the sections.

2. Preliminaries

2.1. Abstract argumentation

An abstract argumentation framework [29] is defined simply as a pair $AF = (AR, att)$ where AR is a set of arguments and $att \subseteq AR \times AR$ and $(A, B) \in att$ means that A *attacks* B.

A set of arguments S attacks (or is attacked by) an argument A (or a set of arguments R) if some argument in S attacks (or is attacked by) A (or some argument in R); S is *conflict-free* if it does not attack itself. A set of arguments S *defends* an argument A if S attacks each argument attacking A.

S is *admissible* if S is conflict-free and defends each argument in it. A *complete extension* is an admissible set of arguments containing each argument it defends. A *preferred extension* is a maximal admissible set of arguments. A *stable extension* is

[5] The paper is both a follow-up and an extension of the papers [25,24]. It is a follow-up of [25,24] as it contains substantially novel results on characterizing canonical attack relations by a least fixed point of an attack-removal function and on the coincidence between canonical and normal attack relations for well-prioritized systems. It is an extension as it offers a much improved presentation as well as detailed proofs of the materials in [25,24].

a conflict-free set of arguments that attacks every argument not belonging to it. It is well-known that both preferred and stable extensions are complete but not vice versa.

The *characteristic function* of AF is defined by

$$F_{AF}(S) = \{A \in AR \mid S \text{ defends } A\}.$$

Since F_{AF} is a monotonic function, there exists a least fixed point of F_{AF}. The grounded extension is defined as the least fixed point of F_{AF}. As complete extensions coincide with conflict-free fixed points of F_{AF}, the grounded extension is also the least complete extension.

2.2. Defeasible knowledge bases

In this section, we recall the basic notions and notations on knowledge bases from [23]. We assume a non-empty set $\mathcal{L}$ of ground atoms (also called a positive literal) and their classical negations (also called negative literals). A set of literals is said to be *contradictory* iff it contains an atom a and its negation $\neg a$. The complement of a positive literal α is $\neg\alpha$ while the complement of a negative literal $\neg\alpha$ is α. Abusing the notation slightly, we denote the complement of any literal λ by $\neg\lambda$.

We distinguish between *domain atoms* representing propositions about the concerned domains and *non-domain atoms* of the form ab_d representing the non-applicability of defeasible rules d (even if the premises of d hold).

We denote by $\mathcal{L}_{dom}$ the set of all domain literals.

We distinguish between strict and defeasible rules as often done in the literature [43,44,37,38,52,23]. A *defeasible* (resp. *strict*) rule r is of the form $b_1, \ldots, b_n \Rightarrow h$ (resp. $b_1, \ldots, b_n \rightarrow h$) where $b_1, \ldots, b_n \in \mathcal{L}_{dom}$ and $h \in \mathcal{L}_{dom}$ or h is an atom of the form ab_d. The set $\{b_1, \ldots, b_n\}$ (resp. h) is referred to as the *body* (resp. *head*) of r and denoted by bd(r) (resp. hd(r)).

Definition 1.

1. A **rule-based system** is a triple $\mathcal{R} = (RS, RD, \preceq)$ where
   (a) RS is a set of strict rules, and
   (b) RD is a set of defeasible rules, and
   (c) for each rule in $RS \cup RD$ whose head is a non-domain atom ab_d, it holds that $d \in RD$, and
   (d) $\preceq$ is a transitive relation over RD representing the preferences between defeasible rules, whose strict core is $\prec$ (i.e. $d \prec d'$ iff $d \preceq d'$ and $d' \npreceq d$ for $d, d' \in RD$).
2. A **knowledge base** is defined as a pair $K = (\mathcal{R}, BE)$ consisting of a rule-based system $\mathcal{R}$, and a set $BE \subseteq \mathcal{L}_{dom}$, the *base of evidence* of K, representing unchallenged observations, facts etc.[6]
   For convenience, knowledge base K is often written directly as a quadruple $(RS, RD, \preceq, BE)$ where the components RS, RD, $\preceq$ or BE are often referred to by $RS_K, RD_K, \preceq_K$ or BE_K respectively.
3. A knowledge base K is ***basic*** if its precedence relation is empty (i.e. $\preceq_K = \emptyset$).

A knowledge base is essentially a defeasible theory in [17] together with a set of preferences between defeasible rules where following Modgil&Prakken [43] we separate the evidence base from the set of rules. The separation is necessary as many key properties of attack relations (like the properties of context-independence (Definition 17) or credulous cumulativity (section 8.2) are defined across distinct knowledge bases with the same set of rules.[7] A knowledge base is an argumentation theory as defined in [43] with only one kind of (classical) negation (another type of negation is added later in section 9).

Definition 2. Let $K = (RS, RD, \preceq, BE)$ be a knowledge base. An **argument** wrt K is a proof tree defined inductively as follows:

1. For each $\alpha \in BE$, $[\alpha]$ is an argument with conclusion α.
2. Let r be a rule of the forms $\alpha_1, \ldots, \alpha_n \rightarrow / \Rightarrow \alpha$, $n \geq 0$, from $RS \cup RD$ and $A_1, \ldots, A_n$ be arguments with conclusions α_i, $1 \leq i \leq n$, respectively. Then $A = [A_1, \ldots, A_n, r]$ is an argument with *conclusion* α and *last rule* r denoted by $cnl(A)$ and $last(A)$ respectively.
   Note that for argument of the form $[\alpha]$, $\alpha \in BE$, $last([\alpha])$ is not defined.
3. Each argument wrt K is obtained by applying the above steps 1, 2 finitely many times.

Example 3. Consider a knowledge base K (adapted from [15,16,23]), consisting of three defeasible rules

[6] In [23], undercut rules (i.e. rules with heads of the form ab_d) are strict rules. Our more recent work on application of structured argumentation [46] suggests that it is more convenient to allow undercut rules to be both strict and defeasible as it is the case in this paper. Note that allowing undercut rules to be both strict and defeasible do not make the system more expressive as each defeasible undercut rule of the form $bd \Rightarrow ab_d$ could be equivalently replaced by two rules $bd \Rightarrow \eta$ and $\eta \rightarrow ab_d$ where η is a new atom not appearing anywhere else.

[7] Example 7 illustrates why separating the evidence base from the set of strict rules is needed.

$$\begin{array}{cccc} A_1 : P & A_2 : T & A_3 : \neg T & A_2' : T \\ \Uparrow d_1 & \Uparrow d_2 & \Uparrow d_3 & \Uparrow d_2 \\ D & P & A & P \\ & \Uparrow d_1 & \uparrow r & \\ & D & D & \end{array}$$

Fig. 4. Dean example.

$$d_1 : Dean \Rightarrow Professor \quad d_2 : Professor \Rightarrow Teach \quad d_3 : Administrator \Rightarrow \neg Teach$$

and two strict rules

$$r : Dean \rightarrow Administrator \quad r' : \neg Administrator \rightarrow \neg Dean$$

together with a precedence relation consisting of just $d_2 \prec d_3$. Suppose we know some dean who is also a professor.

The considered knowledge base is represented by $K = (RS, RD, \preceq, BE)$ with $RS = \{r, r'\}$, $RD = \{d_1, d_2, d_3\}$, $\preceq = \{(d_2, d_3)\}$ and $BE = \{D, P\}$.[8]

Relevant arguments can be found in Fig. 4 where $A_1 = [[D], d_1]$, $A_2 = [A_1, d_2]$, $A_2' = [[P], d_2]$, $A_3 = [[[D], r], d_3]$.

Notation 1.

1. The set of all arguments wrt a knowledge base K is denoted by AR_K. The set of the conclusions of arguments in a set $S \subseteq AR_K$ is denoted by $cnl(S)$.
2. A *strict argument* is an argument containing no defeasible rule. An argument is *defeasible* iff it is not strict. A defeasible argument A is called *basic defeasible* iff $last(A)$ is defeasible.
3. For any argument A, the set of defeasible rules appearing in A is denoted by $dr(A)$. The set of last defeasible rules in A, denoted by $ldr(A)$, is $\{last(A)\}$ if A is basic defeasible, otherwise it is equal $ldr(A_1) \cup \ldots \cup ldr(A_n)$ where $A = [A_1, \ldots, A_n, r]$.
4. An argument B is a *subargument* of an argument A iff $B = A$ or $A = [A_1, \ldots, A_n, r]$ and B is a subargument of some A_i. B is a *proper subargument* of A if B is a subargument of A and $B \neq A$.

Definition 3. Let K be a knowledge base and $X \subseteq \mathcal{L}$ and $l \in \mathcal{L}$. Further let $X_{dom} = X \cap \mathcal{L}_{dom}$.[9]

1. We say that l is strictly derived from X wrt K, denoted by $X \vdash_K l$, iff $l \in X$ or l is the conclusion of an argument constructed according to Definition 2 where in step 1 only elements from X_{dom} are used and in step 2, only the strict rules from K are used.
2. The *closure* of a set $X \subseteq \mathcal{L}$ wrt knowledge base K, denoted by $CN_K(X)$, is defined by $CN_K(X) = \{l \mid X \vdash_K l\}$.
3. X is said to be *closed* wrt K iff $X = CN_K(X)$. X is said to be *inconsistent* wrt K iff its closure $CN_K(X)$ is contradictory. X is *consistent* wrt K iff it is not inconsistent wrt K.
4. K is said to be *consistent* iff its base of evidence BE_K is consistent wrt K.

As the notions of closure, consistency depends only on the set of strict rules in the knowledge base, we often write $X \vdash_{RS} l$ or $l \in CN_{RS}(X)$ for $X \vdash_K l$ or $l \in CN_K(X)$ respectively.

Definition 4. Let $\mathcal{R} = (RS, RD, \preceq)$ be a rule-based system and $K = (\mathcal{R}, BE)$ be a knowledge base.

1. $\mathcal{R}$ and K are said to be closed under transposition [17] iff for each strict rule of the form $b_1, \ldots, b_n \rightarrow h$ in RS s.t. $h \in \mathcal{L}_{dom}$, all the rules of the forms $b_1, \ldots, b_{i-1}, \neg h, b_{i+1}, \ldots, b_n \rightarrow \neg b_i$, $1 \leq i \leq n$, also belong to RS.
2. $\mathcal{R}$ and K are said to be closed under contraposition [47,44] iff for each set $S \subseteq \mathcal{L}_{dom}$, each $\lambda \in \mathcal{L}_{dom}$, if $S \vdash_{RS} \lambda$ then for each $\sigma \in S$, $S \setminus \{\sigma\} \cup \{\neg\lambda\} \vdash_{RS} \neg\sigma$.
3. $\mathcal{R}$ and K are said to satisfy the self-contradiction property [27] iff for each minimal inconsistent set $X \subseteq \mathcal{L}_{dom}$, for each $x \in X$, it holds: $X \vdash_{RS} \neg x$.

Lemma 1. *Let $\mathcal{R}$ be a rule-based system that is closed under transposition or contraposition. Then $\mathcal{R}$ satisfies the property of self-contradiction.*

Proof. A proof is given in [23]. To keep the paper self-contained, we recall the proof in Appendix A. □

[8] D, P, T, A stand for Dean, Professor, Teach and Administrator respectively.

[9] I.e. X_{dom} is X minus the non-domain atoms ab_d.

Definition 5 *(Attack relation).* An *attack relation* for a knowledge base K is a relation $att \subseteq AR_K \times AR_K$ such that there is no attack against strict arguments, i.e. for each strict argument $B \in AR_K$, there is no argument $A \in AR_K$ such that $(A, B) \in att$.

For convenience, we often say A *attacks B wrt att* for $(A, B) \in att$.

2.3. Basic postulates

We recall the postulates of consistency and closure from [17] and of subargument closure from [43,1,42].

Definition 6. Let att be an attack relation for a knowledge base K.

- att is said to satisfy the **consistency postulate** iff for each complete extension E of (AR_K, att), the set $cnl(E)$ of conclusions of arguments in E is consistent.
- att is said to satisfy the **closure postulate** iff for each complete extension E of (AR_K, att), the set $cnl(E)$ of conclusions of arguments in E is closed.
- att is said to satisfy the **subargument closure postulate** iff for each complete extension E of (AR_K, att), E contains all subarguments of its arguments.

For ease of reference, the above three postulates are often referred to as **basic postulates**.

3. Sufficient properties for basic postulates

As the basic postulates are more about the "output" of attack relations rather than about their structure, we present below two simple properties about the structure of attack relation that ensure the satisfaction of the basic postulates. We first recall two key notions of undercut and rebut from the literature [43,45,17].

Definition 7.

- We say A *undercuts* B (at B') iff B' is a basic defeasible subargument of B and $cnl(A) = ab_{last(B')}$.
  We say A *rebuts* B (at B') iff B' is a basic defeasible subargument of B and the conclusions of A and B' are contradictory.
- We say A *directly attacks* B iff A attacks B and A does not attack any proper subargument of B.
- An argument A is said to be *generated by* a set S of arguments iff all basic defeasible subarguments of A are subarguments of arguments in S.

For an illustration of the notion of "being generated by a set of arguments", consider $S = \{B_0, B_1\}$ (see Fig. 5) and A_0. The set of basic defeasible subarguments of A_0 is $\{[d_0]\}$. It is clear that $[d_0]$ is a subargument of B_0. Hence A_0 is generated by S. Similarly, A_1 is also generated by S.

Definition 8 *(Strong subargument structure).*[10] Attack relation att is said to satisfy the *property of strong subargument structure* for K iff for all $A, B \in AR_K$, following conditions hold:

1. A attacks B (wrt att) iff A attacks a basic defeasible subargument of B (wrt att).
2. If A undercuts B then A attacks B wrt att.
3. If A directly attacks B (wrt att) then A undercuts B (at B) or rebuts B (at B).

We present the first result showing that strong subargument property is sufficient to guarantee the postulate of closure.

Lemma 2. *Let att be an attack relation for knowledge base K satisfying the property of strong subargument structure. Further let E be a complete extension of (AR_K, att).*

1. *E contains all arguments generated by E, and*
2. *att satisfies the postulates of closure and subargument closure.*

Proof. Let E' be the set of all arguments generated by E. It is clear that

- $E \subseteq E'$, and
- the sets of basic defeasible subarguments of arguments in E and E' coincide.

[10] The strong subargument structure property is a strengthened combination of two ordinary properties of subargument structure and attack closure in [23] stating that attacks directed against subarguments are attacks against the whole arguments and attacks are based on undercuts and contradictory conclusions.

$$\begin{array}{cccc} A_0 : \neg b & A_1 : \neg a & B_0 : c & B_1 : \neg c \\ \uparrow r_0 & \uparrow r_1 & \uparrow r_2 & \uparrow r_3 \\ a & b & a & b \\ \Uparrow d_0 & \Uparrow d_1 & \Uparrow d_0 & \Uparrow d_1 \end{array}$$

Fig. 5. Generated arguments.

$$\begin{array}{cc} A_0 : a & A_1 : \neg a \\ \Uparrow d_0 & \Uparrow d_1 \end{array}$$

Fig. 6. Effective rebuts.

Therefore from condition 1 in Definition 8, it follows that each attack against E' is also an attack against a basic defeasible subargument of some argument in E and hence an attack against some argument in E. Therefore each attack against E' is counterattacked by E, i.e. E' is defended by E. Since E is complete, E' is a subset of E. Therefore $E' = E$. Therefore $cnl(E)$ is closed and all subarguments of arguments in E belong to E. The lemma holds obviously. □

A set S of arguments of a knowledge base K is said to be *inconsistent* (resp. *consistent*) (wrt K) if the set of the conclusions of its arguments, $cnl(S)$, is inconsistent (resp. consistent) (wrt K). We often ignore K if there are no possibilities for misunderstanding.

We introduce below a new simple property of inconsistency resolving that could be viewed as an adaptation of the consistency-covering property in [27] to structured argumentation systems.

Definition 9 *(Inconsistency resolving).* We say attack relation att satisfies the *inconsistency-resolving property* for K iff for each finite set of arguments $S \subseteq AR_K$, if S is inconsistent then S is attacked (wrt att) by some argument generated by S.

As we will show later, the inconsistency-resolving property is satisfied by common conditions like closure under transposition or contradiction or the property of self-contradiction.

Example 4. Consider the basic knowledge base K consisting of just the rules appearing in arguments in Fig. 5. The set $S = \{B_0, B_1\}$ is inconsistent. The argument A_0 is generated by S. Let $att = \{(X, Y) \mid X$ rebuts Y}. It is obvious that S is attacked by A_0.

We show that att is inconsistency-resolving. Let $\mathcal{A} \subseteq AR_K$. Suppose $\mathcal{A}$ is inconsistent. It is clear that if only one defeasible rule appears in arguments in $\mathcal{A}$ then $\mathcal{A}$ is not inconsistent. Therefore both d_0, d_1 appear in $\mathcal{A}$. Therefore A_0 is generated by $\mathcal{A}$. A_0 rebuts any argument containing d_1. Hence att is inconsistency-resolving.

We present now the first important result.

Theorem 1. *Let att, att' be attack relations for knowledge base K.*

1. *If $att \subseteq att'$ and att is inconsistency-resolving for K then att' is also inconsistency-resolving for K.*
2. *If att satisfies the strong subargument structure and inconsistency-resolving then att satisfies the postulate of consistency.*

Proof. Assertion 1 follows easily from the definition of inconsistency-resolving. We show assertion 2. Suppose E is a complete extension of (AR_K, att). From condition 1 in Lemma 2, each argument generated by E belongs to E. Since att is inconsistency-resolving, if E is inconsistent then E is not conflict-free. Since E is conflict-free, E is hence consistent. □

4. Regular attack relation assignments

In general, attack relations satisfying the basic postulates do not capture the semantics of prioritized rules. To see this point, consider a simple knowledge base consisting of exactly two defeasible rules $d_0 :\Rightarrow a$ and $d_1 \Rightarrow \neg a$ with $d_0 \prec d_1$. There are only two arguments A_0, A_1 as given in Fig. 6.

The attack relation $att = \{(A_0, A_1), (A_1, A_0)\}$ has two preferred (also stable) extensions $E_i = \{A_i\}$, $i = 0, 1$. It is obvious that E_0 satisfies both properties of inconsistency-resolving and strong subargument structure. As the prime purpose of the preference of d_1 over d_0 is to rule out extension E_0, attack relation att does not capture the expected semantics.

Dung [30,23] has proposed several simple properties referred to as ordinary properties, to capture the intuition of prioritized rules. We recall and adapt them below. We also motivate and explain their intuitions. We then present two novel concepts of regular attack relations and regular attack relation assignments that lie at the heart of the semantics of prioritized rules.

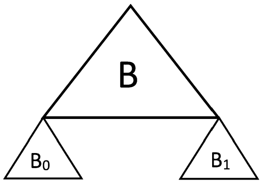

Fig. 7. Modularity of attacks.

4.1. A minimal interpretation of priorities

We first recall from [23] the effective rebut property stating a "minimal interpretation" of a preference $d_0 \prec d_1$ that in situations when both are applicable but accepting both d_0, d_1 is not possible, d_1 should be preferred.

Definition 10 *(Effective rebut).* We say that attack relation *att* satisfies the *effective rebut property* for a knowledge base K iff for all arguments $A_0, A_1 \in AR_K$ such that each A_i, $i = 0, 1$, contains exactly one defeasible rule d_i (i.e. $dr(A_i) = \{d_i\}$), and A_0 rebuts A_1, it holds that A_0 attacks A_1 wrt *att* iff $d_0 \not\prec d_1$.

In Fig. 6, the effective rebut property dictates that A_1 attacks A_0 but not vice versa.

4.2. Propagating attacks

Example 5. Consider the knowledge base in Example 3.

While the effective rebut property determines that A_3 attacks A'_2 (see Fig. 4) but not vice versa (because $d_2 \prec d_3$), it does not say whether A_3 attack A_2.

Looking at the structure of A_2, A'_2, we can say that A_2 is a weakening of A'_2 as the undisputed fact P on which A'_2 is based is replaced by a defeasible belief P (supported by argument A_1). Therefore if A_3 attacks A'_2 then it is natural to expect that A_3 should attack A_2 too.

The above analysis also shows that attacks generated by the effective rebut property, could be propagated to other arguments based on a notion of weakening of arguments. We recall this notion as well as the associated property of attack monotonicity from [23] below.

Let $A, B \in AR_K$ and $AS \subseteq AR_K$. Intuitively, B is a weakening of A by AS if B is obtained by replacing zero, one or more premises of A by arguments in AS whose conclusions coincide with the premises.

Definition 11. Let $A, B \in AR_K$ and $AS \subseteq AR_K$. B is said to be **a weakening of** A by AS iff

1. $A = [\alpha]$ for $\alpha \in BE$, and ($B = [\alpha]$ or $B \in AS$ with $cnl(B) = \alpha$), or
2. $A = [A_1, \ldots, A_n, r]$ and $B = [B_1, \ldots, B_n, r]$ where each B_i is a weakening of A_i by AS.
   By $A \downarrow AS$ we denote the set of all weakenings of A by AS.
   For simplicity, we often say that A is a strengthening of B by AS if B is a weakening of A by AS.

For an illustration, consider again the arguments in Fig. 4. It is clear that $[P] \downarrow \{A_1\} = \{[P], A_1\}$, $A'_2 \downarrow \{A_1\} = \{A'_2, A_2\}$. In Fig. 1, $N_i \downarrow \{[\Rightarrow inno(S)]\} = \{N_i, N'_i\}$, $i = 1, 2$.

The attack monotonicity property states that if an argument A attacks an argument B then A also attacks all weakenings of B. Moreover if a weakening of A attacks B then A also attacks B.

Definition 12 *(Attack monotonicity).* We say attack relation att satisfies the *property of attack monotonicity* for knowledge base K iff for all $A, B \in AR_K$ and for each weakening C of A, for each weakening D of B, the following assertions hold:

1. If $(A, B) \in att$ then $(A, D) \in att$.
2. If $(C, B) \in att$ then $(A, B) \in att$.

We introduce below the property of attack modularity that can be intuitively related to the fact that in a real world conversation, if you claim that my argument is wrong, I would naturally ask which part of my argument is wrong. For an illustration, consider Fig. 7. Suppose C is the argument consisting of B and arguments B_0, B_1 that support some premises of B. It follows that $C \in B \downarrow \{B_0, B_1\}$. Suppose A attacks C. As C is composed by B, B_0, B_1, we would expect that A would attack one of the components B, B_0, B_1 of C. In other words, we expect the attack to be modular in the sense if A attacks an argument then A should attack one of its components. The link-oriented property introduced in [23] represents this intuition. As the formal definition of the "link-orientation" property does not reflect directly the modularity intuition of the property, we introduce an equivalent version of it below and then show their equivalence.

Definition 13 *(Modularity of attacks).* We say that attack relation att satisfies the *property of modularity* for K iff for all arguments $A, B, C \in AR_K$ such that A attacks C where

1. C is a weakening of B by $AS \subseteq AR_K$, and
2. C is not a weakening of B by any proper subset of AS,

then A attacks B or some argument in AS.[11]

We recall the property of link-orientation below and then show its equivalence to the property of modularity of attacks.

Definition 14 *(Link-orientation).* We say that attack relation att satisfies the *property of link-orientation* for K iff for all arguments $A, B, C \in AR_K$ such that C is a weakening of B by $AS \subseteq AR_K$, it holds that if A attacks C (wrt att) and A does not attack any argument in AS (wrt att) then A attacks B (wrt att).

Lemma 3. *An attack relation att satisfies the* property of modularity *for K iff att satisfies the* property of link-orientation *for K.*

Proof. "⇒" Let $A, B, C \in AR_K$ such that C is a weakening of B by $AS \subseteq AR_K$ and A attacks C (wrt att) and A does not attack any argument in AS (wrt att). We show that A attacks B (wrt att).

Let AS_0 be a minimal subset of AS such that C is a weakening of B by AS_0. Since att satisfies the property of modularity for K, it follows that A attacks B or some argument in AS_0. Since A does not attack any argument in AS and $AS_0 \subseteq AS$, A does not attack any argument in AS_0. Since A attacks B or some argument in AS_0, it follows that A attacks B, i.e. the link-orientation is satisfied if modularity holds.

"⇐" Let $AS \subseteq AR_K$ such that $C \in B \downarrow AS$ and C is not a weakening of B by any proper subset of AS. Suppose A attacks C (wrt att). We show that A attacks B or some argument in AS.

If A attacks some argument in AS, we are done. Suppose A does not attack any argument in AS. From the link-orientation property, we have A attacks B (wrt att), i.e. the modularity is satisfied if link-orientation holds. □

Example 6. Consider again arguments in Fig. 4. Suppose d_2 is now preferred to d_3 (i.e. $d_3 \prec d_2$). The effective rebut property dictates that A_3 does not attack A'_2. Does A_3 attack A_2? Suppose A_3 attacks A_2. Since A_3 does not attack A_1 that is a subargument of A_2, we expect that A_3 should attack some other part of A_2. In other words, we expect that A_3 attacks A'_2. But this is a contradiction to the effective rebut property stating that A'_2 attack A_3 but not vice versa. Hence A_3 does not attack A_2.

4.3. Attack relation assignments: propagating attacks across knowledge bases

While the properties of inconsistency-resolving, strong subargument structure, effective rebuts, attack monotonicity and link-orientation are natural and intuitive, they are still not sufficient for determining an intuitive attack relations wrt prioritized rules. The example below illustrates this point.

Example 7. Consider a knowledge base K_0 obtained from knowledge base K in Example 3 by revising the evidence base to $BE = \{D\}$. It is clear that arguments A_1, A_2, A_3 belong to AR_{K_0} while A'_2 is not an argument in AR_{K_0}.

As A'_2 does not belong to AR_{K_0}, the effective rebut property does not "generate" any attacks between arguments in AR_{K_0}. How could we determine the attack relation for K_0?

As both A_2, A_3 belong to both AR_K, AR_{K_0} and the two knowledge bases K_0, K have identical rule-based system, we expect that the attack relations between their common arguments should be identical. In other words, because A_3 attacks A_2 wrt K (see Example 5), A_3 should attack A_2 also wrt K_0. This intuition is captured by the context-independence property [23] linking attack relations between arguments across the boundary of knowledge bases.

The example also indicates that attack relations of knowledge bases with the same rule-based system should be considered together. This motivates the introduction of the attack relation assignment in Definitions 15, 16.

Definition 15. Let $\mathcal{R} = (RS, RD, \preceq)$ be a rule-based system. The class consisting of all consistent knowledge bases of the form $(\mathcal{R}, BE)$ is denoted by $\mathcal{C}_{\mathcal{R}}$.

[11] The second condition ensures that each argument in AS is a subargument of C. Dropping the second condition would give an equivalent and technically slightly simpler version of the modularity though some argument in AS may not be a component of C.

Assumption. From now on, whenever we mention a rule-based system $\mathcal{R}$, we mean a consistent one, i.e. $\mathcal{C}_{\mathcal{R}} \neq \emptyset$.[12]

Definition 16 *(Attack relation assignment).* An *attack relation assignment atts* for a rule-based system $\mathcal{R}$ is a function assigning to each knowledge base $K \in \mathcal{C}_{\mathcal{R}}$ an attack relation $atts(K) \subseteq AR_K \times AR_K$.[13,14]

We next recall the context-independence property from [23] stating that the attack relation between two arguments depends only on the rules appearing in them and their preferences.

Definition 17 *(Context-independence).* We say attack relation assignment atts for a rule-based system $\mathcal{R}$ satisfies the property of *context-independence* iff for any two knowledge bases $K, K' \in \mathcal{C}_{\mathcal{R}}$ and for any two arguments A, B from $AR_K \cap AR_{K'}$, it holds that $(A, B) \in atts(K)$ iff $(A, B) \in atts(K')$.

The context-independence property is commonly accepted in many well-known argument-based systems like the assumption-based framework [12,32] and the ASPIC+ approach [47,43].

Notation 2.

- For ease of reference, we refer to the property of context-independence as well as the properties of inconsistency-resolving, strong subargument structure, effective rebuts, attack monotonicity and link-orientation as **regular properties**.
- Let P be a regular property different to the context-independence one.
  We say an attack relation assignment atts satisfies P iff for each knowledge base $K \in \mathcal{C}_{\mathcal{R}}$, $atts(K)$ satisfies P.

We can now present two novel concepts of weakly regular and regular attack relation assignments.

Definition 18 *(Regular attack relation assignments).*

1. An attack relation assignment *atts* for a rule-based system $\mathcal{R}$ is said to be **weakly regular** iff it satisfies the regular properties of context-independence, strong subargument structure, effective rebuts, attack monotonicity and link-orientation. The set of all weakly regular attack relation assignments for $\mathcal{R}$ is denoted by $WRAA_{\mathcal{R}}$.
2. A weakly regular attack relation assignment *atts* for a rule-based system $\mathcal{R}$ is said to be **regular** iff it satisfies the inconsistency-resolving property.
   The set of all regular attack relation assignments for $\mathcal{R}$ is denoted by $RAA_{\mathcal{R}}$.

It is obvious that $RAA_{\mathcal{R}} \subseteq WRAA_{\mathcal{R}}$ holds.

5. The complete upper-semilattice of regular attack relation assignments

5.1. Preliminaries: complete semilattice

We introduce the concept of semilattice. A partial order[15] $\leq$ on a set S is a *upper-semilattice* (resp. *lower-semilattice*) [21,53] iff each pair of elements x, y from S has a supremum (resp. infimum) wrt $\leq$.

An upper-semilattice (resp. lower-semilattice) is *complete* iff each non-empty subset of S has a supremum (resp. infimum).[16]

It follows immediately that each complete upper (resp. lower) semilattice S has a unique greatest (resp. least) element.[17]

[12] A key postulate for evaluation of the semantics of structured argumentation is the postulate of consistency. That also means that any inconsistent extension is not considered to be meaningful. So if a rule-based system is inconsistent then there exists not even consistent knowledge base wrt it. Hence all extensions for any knowledge base with this rule-based system are inconsistent and therefore not meaningful.

Consider a rule-based system containing two rules $\rightarrow a$ and $\rightarrow \neg a$. Any extension of any knowledge base of this rule-based system is inconsistent. The postulate of consistency is never satisfied for any such knowledge base.

So for a rule-based system to make sense, it should be consistent first, i.e. the knowledge base with this rule-based system and an empty set of evidences must be consistent. Otherwise there is no point to consider it (at least for those agents who consider the postulate of consistency to be relevant).

[13] Note that $atts(K)$ is an attack relation (see Definition 5).

[14] In [23], attack relation assignments are defined for sensible classes of knowledge bases that are unions of classes $\mathcal{C}_{\mathcal{R}}$. All the results in this paper could be straightforwardly generalized for the attack relation assignments defined for sensible classes of knowledge bases (see section 8.2 for more discussion).

[15] A reflexive, transitive and antisymmetric relation.

[16] See [21], page 201.

[17] Note that a complete upper-semilattice (resp. lower-semilattice) is a complete lattice iff the least (resp. greatest) element exists [21,53].

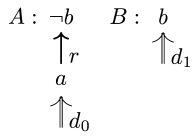

Fig. 8. Non-existence of regular assignments.

5.2. Semilattice structure of regular attack relation assignments

From now on until the end of this section, we assume an arbitrary but fixed rule-based system $\mathcal{R} = (RS, RD, \preceq)$.

Definition 19. Let $\mathcal{A}$ be a non-empty set of attack relation assignments. Define $\bigcup \mathcal{A}$ by:

$$\forall K \in \mathcal{C}_{\mathcal{R}}: \quad (\bigcup \mathcal{A})(K) = \bigcup \{atts(K) \mid atts \in \mathcal{A}\}$$

The following simple lemma and theorem present a deep insight into the structure of regular attack assignments.

Lemma 4. *Let $\mathcal{A}$ be a non-empty set of attack relation assignments.*

1. *Suppose P is a regular property and every attack relation assignment $atts \in \mathcal{A}$ satisfies P. Then $\bigcup \mathcal{A}$ also satisfies P.*
2. *If the attack relation assignments in $\mathcal{A}$ are regular then $\bigcup \mathcal{A}$ is also regular.*
3. *If the attack relation assignments in $\mathcal{A}$ are weakly regular then $\bigcup \mathcal{A}$ is also weakly regular.*

Proof. See Appendix B. $\square$

For attack relation assignments $atts, atts'$, define $atts \subseteq atts'$ iff $\forall K \in \mathcal{C}_{\mathcal{R}}$, $atts(K) \subseteq atts'(K)$.
From Lemma 4, the following Theorem 2 holds obviously.

Theorem 2. *Suppose the set $RAA_{\mathcal{R}}$ of regular attack relation assignments is not empty. Then $(RAA_{\mathcal{R}}, \subseteq)$ is a complete upper-semilattice.* $\square$

Definition 20. Suppose the set $RAA_{\mathcal{R}}$ of all regular attack relation assignments for $\mathcal{R}$ is not empty. The **canonical attack relation assignment** of $\mathcal{R}$ denoted by $Att_{\mathcal{R}}$ is defined by: $Att_{\mathcal{R}} = \bigcup RAA_{\mathcal{R}}$.

Even though in general, regular attack relation assignments (and hence the canonical one) may not exist (as the Example 8 below shows), they exist under natural conditions that we believe most practical rule-based systems satisfy, like the property of self-contradiction or closure under transposition or contraposition as proved in Theorem 3 below.

Example 8. Consider a rule-based system $\mathcal{R}$ consisting of $d_0 :\Rightarrow a \quad d_1 :\Rightarrow b \quad r : a \rightarrow \neg b$ and $d_0 \prec d_1$. Suppose atts be a regular attack relation assignment for $\mathcal{C}_{\mathcal{R}}$. Let $K = (\mathcal{R}, \emptyset)$. The contradicting arguments for K are given in Fig. 8. From the property of effective rebut, it is clear that $(A, B) \notin atts(K)$. Hence $atts(K) = \emptyset$. The inconsistency-resolving property is not satisfied by $atts(K)$, contradicting the assumption that atts is regular. Therefore there exists no regular attack relation assignment for $\mathcal{C}_K$.

While regular attack relation assignments may not exist, weakly regular ones always exist. It turns out that a special type of attack relations, the normal attack relations introduced in [23] are always weakly regular. If the rule-based systems is closed under transposition or contraposition or self-contradiction then normal attack relation assignments are regular.

Definition 21. Let K be a knowledge base and $A, B \in AR_K$.

1. We say that A *normal-rebuts* B (at X) iff A rebuts B (at X) and there is no defeasible rule $d \in ldr(A)$ such that $d \prec last(X)$.
2. The *normal attack relation assignment* $atts_{nr}$ is defined by: For any knowledge base $K \in \mathcal{R}$ and any arguments $A, B \in AR_K$, $(A, B) \in atts_{nr}(K)$ if and only if A undercuts B or A normal-rebuts B.

Before presenting a central result in Theorem 3 below, let us introduce some helpful notations.

Notation 3.

- A *maximal basic defeasible subargument* of B is a basic defeasible subargument of B that is not a proper subargument of any basic defeasible subargument of B.
- A *maximal proper subargument* of B is a proper subargument of B that is not a proper subargument of any proper subargument of B.

Theorem 3.

1. *For any rule-based system $\mathcal{R}$, the normal attack relation assignment $atts_{nr}$ is weakly regular.*
2. *Suppose the rule-based system $\mathcal{R}$ satisfies the self-contradiction property. Then the normal attack relation assignment $atts_{nr}$ is regular and the canonical assignment $Att_{\mathcal{R}}$ exists and $atts_{nr} \subseteq Att_{\mathcal{R}}$.*

Proof. See Appendix B. □

The following lemma follows immediately from the above Theorem 3.

Lemma 5.

1. *$(WRAA_{\mathcal{R}}, \subseteq)$ is a complete upper-semilattice whose supremum is denoted by $Watt_{\mathcal{R}}$.*
2. *If the canonical attack relation assignment $Att_{\mathcal{R}}$ exists, it holds:*

$$Att_{\mathcal{R}} = Watt_{\mathcal{R}}$$

3. *If $\mathcal{R}$ satisfies the self-contradiction property then $Att_{\mathcal{R}}$ exists and $Att_{\mathcal{R}} = Watt_{\mathcal{R}}$.*

Proof. As normal attack relation assignments are weakly regular (Theorem 3). $WRAA_{\mathcal{R}}$ is hence not empty. Assertion 1 follows then from Lemma 4.

We prove assertion 2. If $Att_{\mathcal{R}}$ exists then it is weakly regular and hence $Att_{\mathcal{R}} \subseteq Watt_{\mathcal{R}}$. From the first assertion in Theorem 1, it follows immediately that $Watt_{\mathcal{R}}$ also satisfies the inconsistency-resolving property. Therefore $Watt_{\mathcal{R}}$ is regular. Hence $Watt_{\mathcal{R}} \subseteq Att_{\mathcal{R}}$. Thus $Watt_{\mathcal{R}} = Att_{\mathcal{R}}$.

Assertion 3 follows immediately from second assertion and the second statement in Theorem 3. □

The following lemma follows immediately from the second statement of Theorem 3.

Lemma 6. *Suppose the rule-based system $\mathcal{R}$ satisfies the self-contradiction property. Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$ such that A rebuts B (at B) and $(A, B) \notin Att_{\mathcal{R}}(K)$. Then there is $d \in ldr(A)$ such that $d \prec last(B)$.*

We show below that when all defeasible rules are "equal", i.e. there are no preferences among them, the canonical attack relations coincide with the basic attack relations that are fully determined by undercuts and rebuts. We study further characteristics of canonical attack relations in the following two sections.

Definition 22. The *basic attack relation assignment* for a rule-based system $\mathcal{R}$, denoted by $Batts$ is defined by:

$$\forall K \in \mathcal{C}_{\mathcal{R}}: \; Batts(K) = \{(A, B) \mid A, B \in AR_K, \; A \text{ undercuts or rebuts } B\}$$

Lemma 7.

1. *$Batts$ satisfies all regular properties except the inconsistency-resolving and effective rebut properties.*
2. *If $\mathcal{R}$ satisfies the self-contradiction property then $Batts$ satisfies the inconsistency-resolving property.*
3. *If $\mathcal{R}$ is basic then $Batts$ satisfies the effective rebut property and hence is weakly regular and $Batts = Watt_{\mathcal{R}}$.*

Proof. See Appendix B. □

From Lemma 7, the following theorem holds obviously.

Theorem 4. *If $\mathcal{R}$ is basic and satisfies the self-contradiction property then $Batts$ coincides with the canonical attack relation assignment of $\mathcal{R}$.* □

The following Lemma 8 states that $(RAA_{\mathcal{R}}, \subseteq)$ is not a complete lattice by showing that it is not a lower-semilattice.

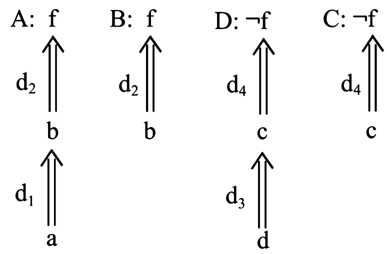

Fig. 9. Infimum of regular assignments.

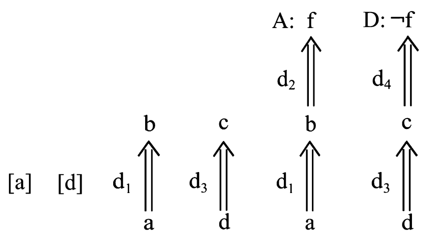

Fig. 10. Inconsistency "Infimum" of regular assignments.

Lemma 8. *In general,* $(RAA_{\mathcal{R}}, \subseteq)$ *is not a lower-semilattice.*

Proof. Consider a rule-based system $\mathcal{R}$ consisting of only four defeasible rules

$$d_1 : a \Rightarrow b \quad d_2 : b \Rightarrow f \quad d_3 : d \Rightarrow c \quad d_4 : c \Rightarrow \neg f$$

and there is no preferences between the defeasible rules.

Let A, B, D, C be the arguments given in Fig. 9.

Define two attack relation assignments $atts, atts'$ for $\mathcal{R}$ as follows.

- For $K \in \mathcal{C}_{\mathcal{R}}$, $atts(K) = Batts(K) \setminus \{(D, A), (D, B)\}$.
- For $K \in \mathcal{C}_{\mathcal{R}}$, $atts'(K) = Batts(K) \setminus \{(A, D), (A, C)\}$.

In Appendix G, we show that both $atts, atts'$ are regular.

We show now that there exists no infimum of $atts, atts'$ in $(RAA_{\mathcal{R}}, \subseteq)$.

Suppose on the contrary the infimum of $atts, atts'$ in $(RAA_{\mathcal{R}}, \subseteq)$ exists. Let it be denoted by $atts_0$. It is clear that for any $K \in \mathcal{C}_{\mathcal{R}}$, $\{(A, D), (D, A)\} \cap atts_0(K) = \emptyset$.

Let $BE_0 = \{a, d\}$ and $K_0 = (\mathcal{R}, BE_0)$.

It is clear that AR_{K_0} consists only of arguments in Fig. 10.

Therefore $atts_0(K_0) = \emptyset$. It is obvious that $atts_0(K_0)$ does not satisfy the inconsistency-resolving property. Thus $atts_0$ is not regular. Contradiction. □

6. Minimal removal semantics: a least-fixed-point characterization of canonical attack relation

From the strong subargument structure property, it is clear that $Att_{\mathcal{R}} \subseteq Batts$. In other words, $\forall K \in \mathcal{C}_{\mathcal{R}}$, the set $Batts(K) \setminus Att_{\mathcal{R}}(K)$ could be viewed as the set of attacks removed from $Batts(K)$ due to the priorities between defeasible rules.

Example 9. For illustration, consider again the rule-based system $\mathcal{R}$ and the associated knowledge base K in Example 3. It is clear that both A_2, A'_2 attack A_3 wrt $Batts(K)$. The introduction of the preference $d_2 \prec d_3$ removed the attack (A'_2, A_3). Since A_2 is a weakening of A'_2, applying the property of attack monotonicity contrapositively, it follows that the attack (A_2, A_3) needs also to be removed.

Consider the knowledge base $K' = (\mathcal{R}, BE')$ where $BE' = \{D\}$. The argument A'_2 does not exist in $AR_{K'}$. Still applying the context-independence property contrapositively, the attack (A_2, A_3) should be removed from the attack relation for K' since it has been removed from the argument system for K.

In general, a contra-positive reading of the regular properties propagates the removal of attacks that are at first removed by the introduction of priorities between defeasible rules.

The discussion illustrates the idea that applying the regular properties in contrapositive forms could be viewed as "attack removal functions". This view leads naturally to a least fixed-point characterization of canonical attack relation assignments. We formalize this insight in this section.

6.1. Preliminaries: complete lattice

A partial order $\le$ on a set S is a **complete lattice** [21] iff each subset X of S has a supremum and infimum wrt $\le$ denoted by $\sqcup X$ and $\sqcap X$ respectively. The greatest and least element of S are often denoted by $\top$, $\bot$ respectively.

A set $X \subseteq S$ is said to be *directed* [21] iff for each pair $x, y \in X$ there exists $z \in X$ such that $x \le z$ and $y \le z$.

Let $(P, \le)$ be a complete lattice. A function $f : P \to P$ is said to be

– *monotone* iff for each $x, y \in P$, if $x \le y$ then $f(x) \le f(y)$;
– *continuous* iff for each directed subset $\Delta \subseteq P$, it holds:

$$f(\sqcup\Delta) = \sqcup f(\Delta)$$

where $f(\Delta) = \{f(x) \mid x \in \Delta\}$.

It is not difficult to see that continuous functions are monotone.

The following fixpoint theorem for continuous function is well-known. To keep the paper self-contained, we also recall the proof.

Theorem 5. *Let $(P, \le)$ be a complete lattice and $f : P \to P$ be continuous. Then f has a least fixpoint, denoted by lfp(f), of the form*

$$lfp(f) = \sqcup_{i=0}^{\infty} f^i(\bot)$$

Proof. It is obvious that $\bot \le f(\bot)$. Therefore $\bot \le f(\bot) \le f^2(\bot) \le \ldots$. Let $\gamma = \sqcup_{i=0}^{\infty} f^i(\bot)$. Since f is continuous, it follows $f(\gamma) = f(\sqcup_{i=0}^{\infty} f^i(\bot)) = \sqcup_{i=0}^{\infty} f(f^i(\bot)) = \sqcup_{i=1}^{\infty} f^i(\bot) = \gamma$. It is obvious that γ is the least fixpoint of f. $\square$

6.2. Least-fixed-point minimal-removal

For $K \in \mathcal{C}_{\mathcal{R}}$, let $REBUT_K$ be the set of all rebuts between arguments in AR_K, i.e.

$$REBUT_K = \{(X, Y) \mid X, Y \in AR_K \text{ and } X \text{ rebuts } Y\}$$

Definition 23. A *removal assignment* is defined as a mapping assigning to each $K \in \mathcal{C}_{\mathcal{R}}$, a subset of $REBUT_K$ representing a set of rebuts that should be removed.

The set of all removal assignments is denoted by $REMAS_{\mathcal{R}}$.

For $\pi, \pi' \in REMAS_{\mathcal{R}}$, define $\pi \subseteq \pi'$ by: $\pi(K) \subseteq \pi'(K)$ for each $K \in \mathcal{C}_{\mathcal{R}}$;
Further let $\mathcal{A} \subseteq REMAS_{\mathcal{R}}$. Define $\bigcup \mathcal{A}$, $\bigcap \mathcal{A}$ by

– $\forall K \in \mathcal{C}_{\mathcal{R}}: \quad (\bigcup \mathcal{A})(K) = \bigcup\{\pi(K) \mid \pi \in \mathcal{A}\}$
– $\forall K \in \mathcal{C}_{\mathcal{R}}: \quad (\bigcap \mathcal{A})(K) = \bigcap\{\pi(K) \mid \pi \in \mathcal{A}\}$

It is obvious that *$(REMAS_{\mathcal{R}}, \subseteq)$ is a complete lattice* where $\bigcup \mathcal{A}$, $\bigcap \mathcal{A}$ represent the supremum and infimum of $\mathcal{A} \subseteq REMAS_{\mathcal{R}}$ wrt $\subseteq$. The infimum of $(REMAS_{\mathcal{R}}, \subseteq)$, that assigns the empty set of rebuts to each knowledge base $K \in \mathcal{C}_{\mathcal{R}}$, is denoted by $\emptyset$.

We introduce below the attack removal functions according to a contra-positive reading of the regular properties.

Let *ER, AM, LO, SA, CI* stand for effective rebuts, attack monotonicity, link-orientation, strong subargument structure and context-independence respectively.

Definition 24. The attack removal functions

$$FER, FAM, FLO, FCI, FSA : REMAS_{\mathcal{R}} \longrightarrow REMAS_{\mathcal{R}}$$

are defined as follows:

Let $K \in \mathcal{C}_{\mathcal{R}}$ and $\pi \in REMAS_{\mathcal{R}}$.

1. $FER(\pi)(K)$ is the set of rebuts that have become ineffective (according to the effective rebut property) due to the introduction of the priorities between defeasible rules, i.e.

   $(A_0, A_1) \in FER(\pi)(K)$ iff $A_0, A_1 \in AR_K$ and each A_i, $i = 0, 1$, contains exactly one defeasible rule d_i and A_0 rebuts A_1 and $d_0 \prec d_1$.

2. $FAM(\pi)(K)$ is the set of rebuts that should be removed as the consequence of the removal of the rebuts in $\pi(K)$ according to a contrapositive reading of the attack monotonicity property, i.e.

$$FAM(\pi)(K) = S_1 \cup S_2$$

where

$$S_1 = \{(A, B) \mid A, B \in AR_K,\ A \text{ rebuts } B \text{ and there is a weakening } B' \text{ of } B \text{ s.t. } (A, B') \in \pi(K)\}$$

$$S_2 = \{(A, B) \mid A, B \in AR_K,\ A \text{ rebuts } B \text{ and there is a strengthening } X \text{ of } A \text{ s.t. } (X, B) \in \pi(K)\}$$

3. $FLO(\pi)(K)$ is the set of rebuts that should be removed as the consequence of the removal of the rebuts in $\pi(K)$ according to a contrapositive reading of the link-orientation property, i.e.

$$\begin{aligned} FLO(\pi)(K) = \{(A, B) \mid A, B \in AR_K,\ & A \text{ rebuts } B \text{ and} \\ & \exists B_0 \in AR_K, AS \subseteq AR_K \text{ such that} \\ & B \in B_0 \downarrow AS, \text{ and} \\ & \text{if } A \text{ rebuts } B_0 \text{ then } (A, B_0) \in \pi(K), \text{ and} \\ & \forall X \in AS: \text{ if } A \text{ rebuts } X \text{ then } (A, X) \in \pi(K)\} \end{aligned}$$

4. $FCI(\pi)(K)$ is the set of rebuts that should be removed as the consequence of the removal of the rebuts in $\pi(K')$ according to a contrapositive reading of the context-independence property, i.e.

$$FCI(\pi)(K) = \{(A, B) \mid A, B \in AR_K, A \text{ rebuts } B \text{ and } \exists K' \in \mathcal{C}_{\mathcal{R}} : (A, B) \in \pi(K')\}$$

5. $FSA(\pi)(K)$ is the set of rebuts that should be removed as the consequence of the removal of the rebuts in $\pi(K)$ according to a contrapositive reading of the strong subargument structure property, i.e.

$$\begin{aligned} FSA(\pi)(K) = \{(A, B) \mid A, B \in AR_K, & A \text{ rebuts } B \text{ and} \\ & \text{for each basic defeasible subargument } X \text{ of } B: \\ & \text{if } A \text{ rebuts } X \text{ (at } X) \text{ then } (A, X) \in \pi(K)\} \end{aligned}$$

Example 10. Continuing Example 9, let $\pi_1 = FER(\overline{\emptyset})$. It holds: $\pi_1(K) = \{(A'_2, A_3)\}$ and $\pi_1(K') = \emptyset$.

Let $\pi_2 = FAM(\pi_1)$. Since A_2 is a weakening of A'_2, and $(A'_2, A_3) \in \pi_1(K)$, it follows directly from the definition of FAM that $(A_2, A_3) \in \pi_2(K)$.

It is not difficult to see that $\pi_2(K') = \emptyset$.

Let $\pi_3 = FCI(\pi_2)$. From $(A_2, A_3) \in \pi_2(K)$, it follows $(A_2, A_3) \in \pi_3(K')$.

Suppose the preference is revised to $d_3 \prec d_2$.

Let $\pi_1 = FER(\overline{\emptyset})$. It holds: $\pi_1(K) = \{(A_3, A'_2)\}$ and $\pi_1(K') = \emptyset$.

Let $\pi_2 = FLO(\pi_1)$. From $A_2 \in A'_2 \downarrow \{A_1\}$ and A_3 rebuts A'_2 and $(A_3, A'_2) \in \pi_1(K)$ and A_3 does not rebut A_1, it follows $(A_3, A_2) \in \pi_2(K)$.

Consider the knowledge base KB in Example 1 (Fig. 1).

Let $\pi_1 = FER(\overline{\emptyset})$. It is clear that $\pi_1(KB) = \{(N_1, A_2), (N_1, N_2)\}$.

It holds that $FSA(\pi_1)(KB) = \{(N_1, A_2), (N_1, N_2), (N_1, N'_2)\}$.

It is easy to see that all functions FER, FAM, FLO, FSA, FCI are monotone.

Lemma 9. *Let $\pi, \pi' \in REMAS_{\mathcal{R}}$ such that $\pi \subseteq \pi'$. Further let $\mathcal{X} \in \{FER, FAM, FLO, FSA, FCI\}$. Then $\mathcal{X}(\pi) \subseteq \mathcal{X}(\pi')$.*

Proof. Follows immediately from Definition 24. □

We define the operator $\sqcup$ on functions of removal assignments. Let $\mathcal{X}, \mathcal{Y} : REMAS_{\mathcal{R}} \longrightarrow REMAS_{\mathcal{R}}$. Define

$$\mathcal{X} \sqcup \mathcal{Y} : REMAS_{\mathcal{R}} \longrightarrow REMAS_{\mathcal{R}}$$

by

$$(\mathcal{X} \sqcup \mathcal{Y})(\pi) = \mathcal{X}(\pi) \cup \mathcal{Y}(\pi)$$

Definition 25. Define

$$REMOVE = FER \sqcup FAM \sqcup FLO \sqcup FSA \sqcup FCI$$

The following Lemma 9 holds obviously:

Lemma 10. *REMOVE is monotone, i.e. for all $\pi, \pi' \in REMAS_{\mathcal{R}}$ such that $\pi \subseteq \pi'$, $REMOVE(\pi) \subseteq REMOVE(\pi')$.*

We show next the continuity of removal functions.

Lemma 11.

1. *All removal functions FER, FAM, FLO, FCI, FSA are continuous, i.e. for each directed $\Delta \subseteq REMAS_{\mathcal{R}}$, for each $\mathcal{X} \in \{FER, FAM, FLO, FCI, FSA\}$, it holds:*

$$\mathcal{X}(\bigcup \Delta) = \bigcup \mathcal{X}(\Delta)$$

2. *REMOVE is continuous and*

$$lfp(REMOVE) = \bigcup_{i=1}^{\infty} REMOVE^i(\overline{\emptyset})$$

Proof. See Appendix C. □

Lemma 12. *$Batts \setminus lfp(REMOVE)$ is weakly regular and coincides with $Watt_{\mathcal{R}}$, i.e.*[18]

$$Watt_{\mathcal{R}} = Batts \setminus lfp(REMOVE)$$

Proof. See Appendix C. □

Theorem 6 below gives a least-fixed point characterization of the canonical attack relation assignment.

Theorem 6.

1. *If the canonical attack relation assignment $Att_{\mathcal{R}}$ exists, it holds:*

$$Att_{\mathcal{R}} = Watt_{\mathcal{R}} = Batts \setminus lfp(REMOVE)$$

2. *If $\mathcal{R}$ satisfies the self-contradiction property then $Att_{\mathcal{R}}$ exists and*

$$Att_{\mathcal{R}} = Watt_{\mathcal{R}} = Batts \setminus lfp(REMOVE)$$

Proof. Follows immediately from Lemmas 5, 12. □

7. Canonical attack relations and normal attack relations

There is a close relationship between canonical and normal attack relations. Even though they are different as illustrated in the following Example 11, they coincide for a relevant class of well-prioritized rule-based systems. Moreover, normal and canonical attack relations are equivalent wrt stable semantics in general.

Example 11. Let $\mathcal{R}$ consist of the following rules:

$$d_0 :\Rightarrow a \qquad d_1 :\Rightarrow b \qquad d_2 : a \Rightarrow c \qquad d_3 : b \Rightarrow \neg c$$

$$r_0 : a \rightarrow \neg b \qquad r_1 : b \rightarrow \neg a$$

and $d_2 \prec d_3$ (a graphical presentation of the rules is given in Fig. 11 where a bar on an arrow indicates that the conclusion of the rule is negated).

Let $K_0 = (\mathcal{R}, \emptyset)$.

Consider the arguments $A_0 = [d_0]$, $A_1 = [d_1]$, $A_2 = [[d_0], d_2]$, $A_3 = [[d_1], d_3]$, $B_0 = [[d_0], r_0]$, $B_1 = [[d_1], r_1]$.

We show that the basic attack relation assignment $Batts$ is also the canonical attack relation assignment. It is clear that $\mathcal{R}$ is closed under transposition. Hence from Lemma 7, it follows that all regular properties except the effective rebut one are satisfied.

Since the set $\{a, b\}$ is inconsistent, there is no knowledge base $K \in \mathcal{C}_{\mathcal{R}}$ such that $\{a, b\} \subseteq BE_K$. Therefore arguments $[[a], d_2]$ and $[[b], d_3]$ never coexist in the same knowledge base. Hence the effective rebut property is always satisfied. Therefore $Batts$ is a regular attack relation assignment for $\mathcal{R}$. $Batts$ is hence also the canonical attack relation assignment.

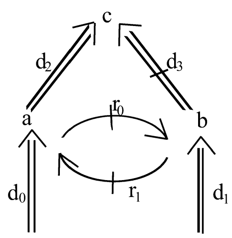

Fig. 11. Rebut-redundance.

It is not difficult to see that for any $K \in \mathcal{C}_{\mathcal{R}}$, $atts_{nr}(K) \subseteq Batts(K) \setminus \{(A_2, A_3)\}$ (Note that for each $K \in \mathcal{C}_{\mathcal{R}}$, $(A_2, A_3) \in Batts(K)$).

Looking closely at A_2, A_3, B_0, B_1, we can say that the rebutting between arguments A_2, A_3 is "redundant" as "their conflict" lies deeper down between B_0, B_1. Resolving the conflict between B_0, B_1 would lead to a resolution of the conflict between A_2, A_3. Introducing a priority between d_2, d_3 is unnecessary and unhelpful. This observation raises two interesting questions:

- When is it helpful and sensible to introduce preferences between defeasible rules?
  We study this question by introducing a class of well-prioritized systems where priorities are sensible. We show that for well-prioritized knowledge bases, the canonical and normal attack relations coincide.
- What could we say about the relationship between canonical and normal attack relations in (possibly not well-prioritized) systems?
  We do not have a comprehensive answer to this question. We will show that in general, for any knowledge base, the canonical and normal attack relations are equivalent wrt stable semantics, i.e. they deliver the same stable extensions. We let it open the question whether the same can be said for other semantics like the preferred extension semantics. In a way, the coincidence of the two attack relations for the class of well-prioritized systems could be viewed as providing a partial answer to this open question.

Before showing the coincidence of normal and canonical attack relations for well-prioritized knowledge bases as well as their equivalence wrt stable semantics, we first present a result to shed further light on the structure of regular attack relations with regards to the strengthening operation.[19]

Let A be a defeasible argument and $d \in ldr(A)$. Let X be a maximal basic defeasible subargument of A whose last link is d.[20] It is clear that X is of the form $X = [X_1, \ldots, X_n, d]$.

Let $Y_1, \ldots, Y_m$ be the maximal basic defeasible subarguments of A that are different to X.[21]

We define a *d-strengthening of A*, denoted by $str(A, d)$ to be a strengthening of A where the subarguments $X_1, \ldots, X_n$, $Y_1, \ldots, Y_m$ are replaced by their conclusions.[22]

Lemma 13. *Let atts be a regular attack relation assignment for $\mathcal{R}$. Further let $K \in \mathcal{C}_{\mathcal{R}}$, $A, B \in AR_K$ and $d \in ldr(A)$ such that the following properties hold:*

- *A attacks B (wrt atts(K)).*
- *A rebuts B (at B) and A does not rebut any proper subargument of B.*
- *Both $str(B, last(B))$ and $str(A, d)$ belong to AR_K.*

The following conditions hold:

1. *$str(A, d)$ attacks $str(B, last(B))$ (wrt atts(K)).*
2. *$d \not\prec last(B)$.*

Proof. See Appendix D. □

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[18] I.e. $Watt_{\mathcal{R}}(K) = Batts(K) \setminus lfp(REMOVE)(K)$ for each $K \in \mathcal{C}_{\mathcal{R}}$.

[19] The readers could skip Lemma 13 below if they are not interested in the proofs of the following Lemmas 14, 18.

[20] Note that a maximal basic defeasible subarguments of A is a basic defeasible subargument of A that is not a proper subargument of any basic defeasible subargument of A.

[21] It is easy to see that if A is basic defeasible then $X = A$, $d = last(A)$ and $m = 0$.

[22] It is clear that str(A, d) contains exactly one defeasible rule that is d.

7.1. Well-prioritized rule-based systems

For this section, let $\mathcal{R} = (RS, RD, \preceq)$ be an arbitrary but fixed rule-based system. Further let $\lambda, \beta \in \mathcal{L}$.

We say that λ *directly depends* on β iff there is a rule $r \in RS \cup RD$ such that $\lambda = hd(r)$ and $\beta \in bd(r)$.

λ *depends* on β iff $\lambda = \beta$ or λ depends on α that directly depends on β.

The set of all sentences in $\mathcal{L}$ on which λ depends is denoted by $\Delta(\lambda)$. For a set $S \subseteq \mathcal{L}$, $\Delta(S)$ is the union of $\Delta(\lambda)$ for $\lambda \in S$.

Definition 26 *(Well-prioritized systems).* A rule-based system $\mathcal{R} = (RS, RD, \preceq)$ is said to be well-prioritized iff for each defeasible rule $d \in RD$, the following condition holds:

If there exists $d' \prec d$ then the set $\Delta(bd(d)) \cup \Delta(\neg hd(d))$ is consistent.

Example 12.

- Consider defeasible rule d_3 in Example 11 again. Since $\Delta(bd(d_3)) \cup \Delta(\neg hd(d_3)) = \Delta(b) \cup \Delta(c) = \{a, b, c\}$ is inconsistent and $d_2 \prec d_3$, the rule-based system not well-prioritized.
- Consider rule d_2 in Example 1. $\Delta(bd(d_2)) \cup \Delta(\neg hd(d_2)) = \Delta(\neg Inno(P_2)) = \{\neg Inno(P_2), Inno(P_1), Inno(S)\}$ is consistent. Since $d_1 \prec d_2$ is the only preference, the concerned rule-based system is well-prioritized.
- Consider rule d_3 in Example 3. $\Delta(bd(d_3)) \cup \Delta(\neg hd(d_3)) = \Delta(A) \cup \Delta(T) = \{A, D, T, P\}$ is consistent. Since $d_2 \prec d_3$ is the only preference, the concerned rule-based system is well-prioritized.

To see the intuition of the idea of well-prioritized systems, let us remember that the "minimal interpretation" of a preference $d' \prec d$ is that in situations when both d', d are applicable but accepting both d', d is not possible, d should be preferred. In other words, in situations where d' is not applicable, a preference $d' \prec d$ is redundant and unnecessary.

For simplicity, let us assume that the heads of d', d are contrary.

It is obvious that in situation when $bd(d') \cup bd(d)$ hold, but $bd(d') \cup bd(d) \cup \{hd(d')\} = bd(d') \cup bd(d) \cup \{\neg hd(d)\}$ is inconsistent, d' would not be applicable. Hence imposing a preference of d over d' is unnecessary and unhelpful in this case.

Therefore we can say that imposing a preference of defeasible rule d over some defeasible rule d' would be only helpful and sensible if $bd(d') \cup bd(d) \cup \{\neg hd(d)\}$ is consistent.

From $bd(d') \cup bd(d) \cup \{\neg hd(d)\} \subseteq \Delta(bd(d)) \cup \Delta(\neg hd(d))$, we can say that in well-prioritized systems, preferences between defeasible rules are sensible.

Before showing the coincidence of normal and canonical attack relations for well-prioritized knowledge bases, we first show a lemma stating in essence that for well-prioritized systems, the direct attacks wrt normal and canonical attack relations coincide.

Lemma 14. *Suppose $\mathcal{R}$ be a well-prioritized rule-based system such that the canonical attack relation assignment $Att_{\mathcal{R}}$ exists. Further let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$ and $d \in ldr(A)$ such that A rebuts B (at B) and $d \prec last(B)$. Then $(A, B) \notin Att_{\mathcal{R}}(K)$.*

Proof. See Appendix D. □

From Lemma 6 and the above Lemma 14, it follows immediately

Lemma 15. *Suppose $\mathcal{R}$ be a well-prioritized rule-based system satisfying the self-contradiction property. Further let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$ such that A rebuts B (at B). It holds that A does not attack B wrt $Att_{\mathcal{R}}(K)$ iff there exists $d \in ldr(A)$ s.t. $d \prec last(B)$.*

We can now prove the coincidence of canonical and normal attack relations for well-prioritized systems.

Theorem 7. *Let $\mathcal{R}$ be a well-prioritized rule-based system satisfying the self-contradiction property. The canonical attack relation assignment $Att_{\mathcal{R}}$ and the normal attack relation assignment $atts_{nr}$ coincide.*

Proof. Because $atts_{nr} \subseteq Att_{\mathcal{R}}$, we only need to show that for each $K \in \mathcal{C}_{\mathcal{R}}$, if $(A, B) \notin atts_{nr}(K)$ then $(A, B) \notin Att_{\mathcal{R}}(K)$.

Let $K \in \mathcal{C}_{\mathcal{R}}$ and $(A, B) \notin atts_{nr}(K)$. It is clear that A does not undercut B. If A does not rebut B then the theorem holds obviously.

Suppose A rebuts B. Therefore for all basic defeasible subarguments X of B, if A rebuts X (at X) then $\exists d_X \in ldr(A)$ s.t. $d_X \prec last(X)$. From Lemma 15, it holds that for all basic defeasible subarguments X of B, if A rebuts X (at X) then A does not attack X wrt $Att_{\mathcal{R}}(K)$.

We show that A does not attack B wrt $Att_{\mathcal{R}}(K)$. Suppose the contrary that A attacks B wrt $Att_{\mathcal{R}}(K)$. Since $Att_{\mathcal{R}}$ is regular, it satisfies the strong subargument structure property. Let X_0 be a basic defeasible subargument of B s.t. A directly attacks X_0 (wrt $Att_{\mathcal{R}}$). As A rebuts B, it follows A rebuts X_0 (at X_0). Therefore A does not attack X_0 wrt $Att_{\mathcal{R}}(K)$. Contradiction. □

It is not difficult to see that checking the well-prioritizedness of a rule-based system is polynomial wrt number n of rules in it.

1. For each $\lambda \in \mathcal{L}$, computing $\Delta(\lambda)$ is polynomial as the computation of $\Delta(\lambda)$ could be done with maximal n iterations to check which rules could be backward-applied to generate $\Delta(\lambda)$ and once such a rule is found, it will be removed from the set of rules that have not been applied yet.
2. For any defeasible rule d, checking whether $\Delta(\neg hd(d)) \cup \Delta(bd(d))$ is consistent is polynomial as it could be done in two steps:
   (a) Compute $\Delta(\neg hd(d)) \cup \Delta(bd(d))$.
   (b) Compute the closure $\Delta(\neg hd(d)) \cup \Delta(bd(d))$. This step is polynomial as it could be done with maximal n iterations to check which strict rules could be forward-applied and once such a rule is found, it will be removed from the set of rules that have not been applied yet.
3. For any defeasible rule d, checking whether there exists $d' \prec d$ is linear.

7.2. Equivalence of canonical and normal attack relations wrt stable semantics

Even though in general canonical attack relations and normal attack relations are different, they turn out to be equivalent for stable semantics in the sense that their stable extensions coincide as we will prove shortly below.

We first introduce some useful lemmas.

Lemma 16. *Let $atts, atts'$ be regular attack relation assignments for $\mathcal{R}$. Further let $K \in \mathcal{C}_{\mathcal{R}}$ and $S \subseteq AR_K$ such that S contains all arguments generated from its arguments. Then S is conflict-free wrt $atts(K)$ iff S is also conflict-free wrt $atts'(K)$.*

Proof. Suppose S is conflict-free wrt $atts(K)$ and S is not conflict-free wrt $atts'(K)$. Since $atts(K)$ and $atts'(K)$ have the same set of undercuts, some argument in S rebuts another. Since all subarguments of arguments in S belong to S, S is inconsistent. Because $atts(K)$ satisfies the inconsistency-resolving property, some argument in S is attacked (wrt atts(K)) by some generated by S. Since S contains all arguments generated from its arguments, S is not conflict-free wrt atts(K). Contradiction. Therefore S is also conflict-free wrt $atts'(K)$. □

Lemma 17. *Let $atts, atts'$ be regular attack relation assignments for $\mathcal{R}$ such that $atts \subseteq atts'$. Then*

1. *each stable extension of $(AR_K, atts(K))$ is a stable extension of $(AR_K, atts'(K))$; and*
2. *each stable extension of $(AR_K, atts(K))$ is a stable extension of $(AR_K, Att_{\mathcal{R}}(K))$.*

Proof. It is clear that assertion (2) follows immediately from assertion (1) and the definition of $Att_{\mathcal{R}}$. We hence need only to prove assertion (1).

Let E be a stable extension of $(AR_K, atts(K))$. From Lemmas 2 and 16, it is clear that E is also conflict-free wrt $atts'(K)$. As E attacks each argument in $AR_K \setminus E$ wrt $atts(K)$ and $atts(K) \subseteq atts'(K)$, it is obvious that E attacks each argument in $AR_K \setminus E$ wrt $atts'(K)$. E is hence stable wrt $atts'(K)$. □

Lemma 18. *Let $\mathcal{R}$ be a rule-based system satisfying the self-contradiction property and atts be a regular attack relation assignment of $\mathcal{R}$. Further let $K \in \mathcal{C}_{\mathcal{R}}$. Each stable extension of $(AR_K, atts(K))$ is also a stable extension of $(AR_K, atts_{nr}(K))$.*

Proof. See Appendix D. □

Theorem 8. *Suppose the rule-based system $\mathcal{R}$ satisfies the property of self-contradiction. Then for each $K \in \mathcal{C}_{\mathcal{R}}$, each stable extension wrt $atts_{nr}(K)$ is also a stable extension wrt $Att_{\mathcal{R}}(K)$ and vice versa.*

Proof. From Theorem 3, the canonical attack relation assignment $Att_{\mathcal{R}}$ exists. From Lemma 18, every stable extension wrt $Att_{\mathcal{R}}(K)$ is a stable extension wrt $atts_{nr}(K)$. Since $atts_{nr}(K) \subseteq Att_{\mathcal{R}}(K)$, every stable extension wrt $atts_{nr}(K)$ is a stable extension wrt $Att_{\mathcal{R}}(K)$ (Lemma 17). □

8. Discussion

8.1. The lower-semilattice structure of value-based semantics

The value-based approaches to argumentation [4,10,47,43,44] define the semantics of defeasible knowledge bases by first defining a preference relation between arguments and then using the preference relation to define attack relation between arguments. We show in this section that the preference relations between arguments have a complete lower-semilattice structure and hence a least preference relation exists that captures the canonical semantics.

We first introduce a new operator about a "structured intersection" of relations that is needed to characterize the structure of preference relations between arguments.

Any relation $R \subseteq X \times X$ over a set X could be partitioned into a **strict core**, denoted by R_{st} and **symmetric core**, denoted by R_{sy} as follows: $R = R_{st} \cup R_{sy}$ where $R_{st} = \{(a,b) \in R \mid (b,a) \notin R\}$ and $R_{sy} = \{(a,b) \in R \mid (b,a) \in R\}$.

For any relations $R, R' \subseteq X \times X$, we introduce a "**strong intersection**"-operator $R \sqcap R'$ by: $R \sqcap R' = (R_{st} \cap R'_{st}) \cup (R_{sy} \cap R'_{sy})$.

Further, we define a partial order $R \ll R'$ by: $R \ll R'$ iff $R_{st} \subseteq R'_{st}$ and $R_{sy} \subseteq R'_{sy}$.

Definition 27. An **argument preference assignment** (or **ap-assignment** for short) for a rule-based system $\mathcal{R}$ is a function Γ assigning to each knowledge base $K \in \mathcal{C}_{\mathcal{R}}$, a relation $\sqsubseteq_{\Gamma,K} \subseteq AR_K \times AR_K$ (whose strict core is $\sqsubset_{\Gamma,K}$) representing a preference relation between arguments in AR_K where strict arguments are not strictly less preferred than any other arguments.

For ap-assignments Γ_0, Γ_1, we write $\Gamma_0 \ll \Gamma_1$ iff for each $K \in \mathcal{C}_{\mathcal{R}}$, $\Gamma_0(K) \ll \Gamma_1(K)$.

We recall the definition of attack relations from the argument preferences [43,44] below.

Definition 28. Let Γ be an ap-assignment defined for $\mathcal{R}$. The **attack relation assignment derived from** Γ and denoted by atts_{Γ}, is defined by: For each $K \in \mathcal{C}_{\mathcal{R}}$ and all $A, B \in AR_K$, $(A,B) \in atts_{\Gamma}(K)$ iff A undercuts B or A rebuts B (at B') and $A \not\sqsubset_{\Gamma,K} B'$.

It is easy to see that the following lemma holds.

Lemma 19. *Let Γ_0, Γ_1 be ap-assignments defined for $\mathcal{R}$ such that $\Gamma_0 \ll \Gamma_1$. It holds $atts_{\Gamma_1} \subseteq atts_{\Gamma_0}$.*

Proof. Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$ such that $(A,B) \in atts_{\Gamma_1}(K)$.

If A undercuts B then it is obvious that $(A,B) \in atts_{\Gamma_0}(K)$.

Suppose A rebuts B (at B') and $A \not\sqsubset_{\Gamma_1,K} B'$. Since $\Gamma_0 \ll \Gamma_1$ it holds that $\sqsubseteq_{\Gamma_0,K} \ll \sqsubseteq_{\Gamma_1,K}$. Therefore $\sqsubset_{\Gamma_0,K} \subseteq \sqsubset_{\Gamma_1,K}$. Therefore $A \not\sqsubset_{\Gamma_1,K} B'$ implies $A \not\sqsubset_{\Gamma_0,K} B'$. Since A rebuts B (at B') and $A \not\sqsubset_{\Gamma_0,K} B'$, it follows that $(A,B) \in atts_{\Gamma_0}(K)$. □

Definition 29. An ap-assignment Γ is **regular** for $\mathcal{R}$ iff its derived attack relation assignment $atts_{\Gamma}$ is regular.

The *set of all regular ap-assignments* for $\mathcal{R}$ is denoted by $AP_{\mathcal{R}}$.

Notation 4. The "strong intersection"-operator is extended to non-empty set $\mathcal{P}$ of ap-assignments and denoted by $\sqcap\mathcal{P}$ as follows: $(\sqcap\mathcal{P})(K) = \sqcap\{\Gamma(K) \mid \Gamma \in \mathcal{P}\}$.

The following lemma shows that the "strong intersection" forms an infimum operation for regular ap-assignments.

Lemma 20. *Let $\mathcal{P}$ be a non-empty set of regular apr-assignments for $\mathcal{R}$. Then $\sqcap\mathcal{P}$ is regular.*

Proof. See Appendix E. □

Theorem 9.

1. *If $AP_{\mathcal{R}}$ is non-empty then $(AP_{\mathcal{R}}, \ll, \sqcap)$ forms a complete lower-semilattice with $CA_{\mathcal{R}} = \sqcap AP_{\mathcal{R}}$ being the least regular ap-assignment for $\mathcal{R}$ and is referred to as the* **canonical ap-assignment**.
2. *If the rule-based system $\mathcal{R}$ satisfies the self-contradiction property then the set of regular ap-assignments $AP_{\mathcal{R}}$ is not empty and $atts_{CA_{\mathcal{R}}} = Att_{\mathcal{R}}$.*

Proof. See Appendix E. □

One of the arguably most influential representatives of the value-based approach could be said to be ASPIC+ [43,44] where four ap-assignments are introduced and studied.

Definition 30. [43,44] Let K be a knowledge base and $\preceq$ be the preference relation over defeasible rules of K and Δ, Δ' be two finite sets of defeasible rules of K and $y \in \{E, D\}$,[23] define:

$\Delta \trianglelefteq_y \Delta'$ iff $\Delta \neq \emptyset$ and one of the following conditions holds:

[23] E, D stand for Elitist and Democratic respectively.

1. $\Delta' = \emptyset$.
2. $y = E$ and $\exists d \in \Delta$ s.t. $\forall d' \in \Delta' : d \preceq d'$.
3. $y = D$ and $\forall d \in \Delta\ \exists d' \in \Delta' : d \preceq d'$.

$\lhd_y$ denotes the strict core of $\unlhd_y$.

Definition 31. [43,44] Let K be a knowledge base, A, B be two arguments in AR_K and $y \in \{E, D\}$.

1. B is **preferred to** A according to the **last link principle** and the **y-ordering** (or **y-principle**), denoted by $A \sqsubseteq_{ly} B$ if and only if $ldr(A) \unlhd_y ldr(B)$.
2. B is **preferred to** A according to the **weakest link principle** and the **y-ordering** (or **y-principle**), denoted by $A \sqsubseteq_{wy} B$ if and only if $dr(A) \unlhd_y dr(B)$.

As we have mentioned in Example 1, attack relations derived from ASPIC+ ap-assignments based on democratic principle do not satisfy the attack monotonicity property. To see this point, consider arguments N_1, N_1' and A_2 in Fig. 1. It is not difficult to see that $N_1 \lhd_{xD} A_2$, but $N_1' \not\lhd_{xD} A_2$ for both $x = l$ and $x = w$. Therefore, with respect to the democratic principle, N_1' attacks A_2, but N_1 does not attack A_2. A clear violation of the property of attack monotonicity. In other words, ASPIC+ ap-assignments based on democratic principle are not regular. This helps in explaining why applying the democratic principle leads to semantics that are counter-intuitive to commonsense in Example 1 as we have discussed before.

The following example taken from [23] shows that ASPIC+ ap-assignments based on elitist principle are not regular because their derived attack relations do not satisfy the consistency postulate.

Example 13. Consider the knowledge base K consisting of

1. an empty base of evidence, and
2. four strict rules

$$r_1 : a_2, a_3, a_4 \to \neg a_1 \quad \ldots\ldots\ldots \quad r_4 : a_1, a_2, a_3 \to \neg a_4$$

together with four defeasible rules

$$d_i :\Rightarrow a_i, \qquad 1 \le i \le 4$$

and
3.

$$\preceq = \{d_1, d_2\} \times \{d_1, d_2\} \cup \{d_3, d_4\} \times \{d_3, d_4\}$$

It is clear that $\preceq$ is a preorder and the knowledge base is consistent and closed under transposition.
There are in total 8 arguments:

$$A_i \equiv [\Rightarrow a_i], \qquad 1 \le i \le 4$$

and

$$B_1 \equiv [A_2, A_3, A_4 \to \neg a_1], \ldots\ldots, B_4 \equiv [A_1, A_2, A_3 \to \neg a_4]$$

We first show

$$\{d_1, d_3, d_4\} \lhd_E \{d_2\}$$

From $d_1 \preceq d_2$, it is clear that $\{d_1, d_3, d_4\} \unlhd_E \{d_2\}$. From $d_2 \not\preceq d_3$, it is obvious that $\{d_2\} \not\unlhd_E \{d_1, d_3, d_4\}$.
Similarly, it holds:

$$\{d_2, d_3, d_4\} \lhd_E \{d_1\} \quad \{d_1, d_2, d_3\} \lhd_E \{d_4\} \quad \{d_1, d_2, d_4\} \lhd_E \{d_3\}$$

Therefore for $1 \le i \le 4$, B_i does not attack A_i according to the ASPIC+ argument preferences based on the elitist principle. Therefore both attack relations based on ASPIC+ elitist ap-assignments equal the empty set. It is obvious that the grounded extension (that is also stable) is $\{A_1, \ldots, A_4, B_1, \ldots, B_4\}$ whose set of conclusions $\{a_1, \ldots, a_4, \neg a_1, \ldots, \neg a_4\}$ is obviously inconsistent. Hence the consistency postulate is violated.[24]

[24] The readers are referred to [23] for further discussion.

8.2. Ordinary attack relations

A class of ordinary attack relation assignments defined on sensible classes of knowledge bases as well as their relationship to other approaches on reasoning with prioritized rules has been studied in [30,23].

Formally, a class $\mathcal{K}$ of knowledge bases [23] is said to be sensible iff $\mathcal{K}$ is not empty, and every knowledge base in $\mathcal{K}$ is consistent, and for each knowledge base $K = (\mathcal{R}, BE)$ belonging to $\mathcal{K}$, all consistent knowledge bases of the form $(\mathcal{R}, BE')$ also belong to $\mathcal{K}$.

Let $\mathcal{K}$ be a sensible class of knowledge bases. Further let CR be the set of all rule-based systems of knowledge bases in $\mathcal{K}$. It is not difficult to see that $\mathcal{K} = \bigcup\{\mathcal{C}_{\mathcal{R}} \mid \mathcal{R} \in CR\}$.

It follows immediately that for each consistent rule-based system $\mathcal{R}$, $\mathcal{C}_{\mathcal{R}}$ is a sensible class of knowledge bases.

To relate regular attack relations to ordinary ones, we will first consider only ordinary attack relation assignments defined on $\mathcal{C}_{\mathcal{R}}$. We will discuss the situations when attack relation assignments should be considered wrt sensible classes of knowledge bases at the end of this section.

An attack relation assignment att (defined for $\mathcal{C}_{\mathcal{R}}$) is ordinary [23] if it is weakly regular and also satisfies the important property of credulous cumulativity stating intuitively that if some beliefs in your belief set are confirmed in the reality then your belief set will not change because of it. Credulous cumulativity is a key property satisfied by many argument-based and non-argument-based approaches to reasoning with prioritized rules [30,23]. We give the formal definition of credulous cumulativity below.

A set $S \subseteq \mathcal{L}$ is said to be a *belief set* of knowledge base K wrt an attack relation assignment atts iff there is a stable extension E of $(AR_K, atts(K))$ such that $S = cnl(E)$.

Definition 32 *(Credulous cumulativity).* [23] We say attack relation assignment *atts* satisfies the *property of credulous cumulativity* if and only if for each $K \in \mathcal{C}_{\mathcal{R}}$, for each belief set S of K wrt *atts* and for each finite subset $\Omega \subseteq S$ of domain literals, S is a belief set of $K + \Omega = (RS_K, RD_K, \preceq_K, BE_K \cup \Omega)$ wrt *atts*.

For an illustration, consider again Example 3. Let $K' = (\mathcal{R}, BE')$ where $BE' = \{D\}$. As $\{D, P, A, \neg T\}$ is a belief set of K', the property of credulous cumulativity dictates that $\{D, P, A, \neg T\}$ is also a belief set of $K = K' + \{P\} = (RS_K, RD_K, \preceq_K, \{D, P\})$.

We show below that regular attack relation assignments are ordinary by showing that they satisfy the credulous cumulativity property.

Theorem 10. *The credulous cumulativity property is satisfied by all regular attack relation assignments.*

Proof. See Appendix E. $\square$

In [23], a natural property of irrelevance of redundant defaults stating that adding a defeasible rule of the form $\Rightarrow f$ to a knowledge base K for any evidence $f \in BE_K$ should not change its semantics is presented and shown that it is satisfied by attack relation assignments satisfying the properties of attack monotonicity and context-independence. Adding rules to knowledge bases change their underlining rule-based systems. Therefore attack relation assignments for sensible classes of knowledge bases should be considered when studying the semantics of knowledge base revisions involving adding or deleting rules from the rule-based systems.

In this paper we do not discuss the problems concerning revisions of knowledge bases involving adding or deleting rules from the rule-based systems. Hence we could safely restrict our consideration on sensible classes of knowledge bases sharing the same rule-based system.

8.3. Remark on the underlying domain language $\mathcal{L}_{dom}$

We assume until now that the underlying language of knowledge bases consists only of literals. In contrast, the underlying language of ASPIC+ systems [43] could be any logical language. Similarly, assumption-based systems [31,32] do not put any restriction on their underlying language.

From our own experiences in research on application of structured argumentation [33,28,26,40,46], we believe that many applications will be based on language of literals. Hence, from a pragmatic point of view, it is sensible to focus on an underlying language of literals first.

Anyway, readers who prefer to work with a general underlying language, could simply assume that the domain language $\mathcal{L}_{dom}$ be any logical language containing a classical negation operator $\neg$ (note that non-domain atoms of the form ab_d do not belong to $\mathcal{L}_{dom}$). In all definitions, notations, lemmas and theorems in this paper starting from Definition 1 up to the last definition, references are always made to $\mathcal{L}_{dom}$, and not to a specific language of literals. Hence all theorems and lemmas in this paper are also correct with respect to a general language $\mathcal{L}_{dom}$. More formally, we extend in the section 9 our framework to allow a general language $\mathcal{L}_{dom}$ together with negation-as-failure assumptions.

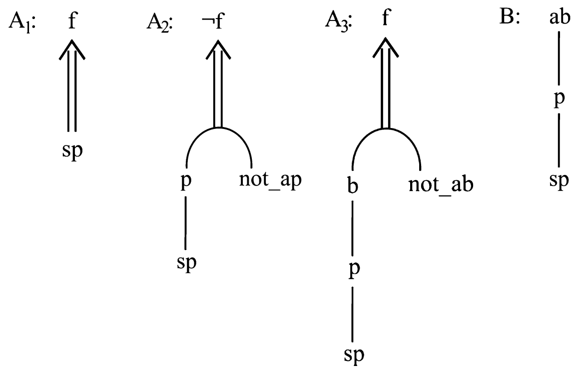

Fig. 12. Extended knowledge base.

9. Extending knowledge bases with assumptions

Assumption-based argumentation [31,32] is a well-known approach to structured argumentation based on the concept that assumptions are acceptable if there is no evidence to the contrary. We show in this section that assumptions could naturally be incorporated into our framework.

Let $\mathcal{L}_{dom}$ be a logical language possibly containing a classical negation operator $\neg$. Further $\mathcal{L}$ be the language consisting of sentences in $\mathcal{L}_{dom}$ and non-domain atoms of the form ab_d.

Definition 33.

1. An *extended rule-based system* over $\mathcal{L}$ is a triple $(\mathcal{R}, \mathcal{A}, \overline{})$ where
   (a) $\mathcal{R} = (RS, RD, \preceq)$ is a rule-based system over $\mathcal{L}$, and
   (b) $\mathcal{A} \subseteq \mathcal{L}_{dom}$ is a set of assumptions such that assumptions in $\mathcal{A}$ do not appear in the heads of rules in $\mathcal{R}$, and
   (c) $\overline{}$ is a (total) one-one mapping from $\mathcal{A}$ into $\mathcal{L}_{dom}$, where $\overline{x}$ is referred to as the *contrary* of x.
2. An *extended knowledge base* consists of an extended rule-based system $(\mathcal{R}, \mathcal{A}, \overline{})$ and a set of evidences $BE \subseteq \mathcal{L}_{dom}$ disjoint to $\mathcal{A}$.

Remark 1. Abusing slightly the notation for simplicity, we often denote an extended rule-based system $(\mathcal{R}, \mathcal{A}, \overline{})$ simply by the pair $(\mathcal{R}, \mathcal{A})$ or just $\mathcal{R}$ if no misunderstanding is possible.

Similarly, an extended knowledge base $(\mathcal{R}, \mathcal{A}, \overline{}, BE)$ is often denoted by the triple $(\mathcal{R}, \mathcal{A}, BE)$ or just the pair $(\mathcal{R}, BE)$ if no misunderstanding is possible.

Remark 2. Extended knowledge bases could also be viewed as extended assumption-based systems where defeasible rules together with two new types of attacks, rebuts and undercuts, are added. As illustrated in the following example, such additions could provide a more flexible platform for developers for representing their domains.

Example 14. An extended knowledge base $(RS, RD, \preceq, BE)$ representing the well-known example of "penguins don't fly while birds and super-penguins do" is given by:

- RS consisting of the strict rules

$$sp \to p, \qquad p \to b, \qquad p \to ab$$

  stating that super-penguins are penguins, penguins are birds and penguins are abnormal birds, and
- RD containing three defeasible rules:

$$sp \Rightarrow f, \qquad not\_ap, p \Rightarrow \neg f, \qquad not\_ab, b \Rightarrow f$$

  stating that super-penguins fly by default while penguin normally do not fly and birds normally fly.
- $\preceq$ consisting of an unique preference

$$(not\_ap, p \Rightarrow \neg f) \preceq (sp \Rightarrow f)$$

  stating that the rule "super-penguins fly (by default)" has higher priority than the rule "penguins normally do not fly", and
- $BE = \{sp\}$.

The relevant arguments are given in Fig. 12.

Notation 5 *(Adaptations for extended knowledge bases).*

1. Arguments wrt extended knowledge base $K = (\mathcal{R}, \mathcal{A}, BE)$ are defined as in Definition 2 where condition 1 is revised as follows:

   For each $\alpha \in BE \cup \mathcal{A}$, $[\alpha]$ is an argument with conclusion α

   Arguments of the form $[\alpha]$, $\alpha \in \mathcal{A}$, are also called *assumption-arguments*.
2. A **strict argument** is an argument containing no defeasible rule and no assumptions. An argument is **defeasible** iff the set of defeasible rules of A is not empty, i.e. $ldr(A) \neq \emptyset$. A defeasible argument A is called **basic defeasible** iff $last(A)$ is defeasible. An argument is **non-defeasible** iff it is not defeasible.
   Note that a non-strict argument could also be non-defeasible if it contains some assumptions but no defeasible rules.
3. Let $X \subseteq \mathcal{L}$ and $l \in \mathcal{L}$. Further let $X_{dom} = X \cap \mathcal{L}_{dom}$.
   We say that l is *strictly derived* from X wrt K, denoted by $X \vdash_K l$, iff $l \in X$ or l is the conclusion of an argument constructed using only elements from X_{dom} and the strict rules from K.[25]
   The **closure** of a set $X \subseteq \mathcal{L}$ wrt knowledge base K, denoted by $CN_K(X)$, is defined by $CN_K(X) = \{l \mid X \vdash_K l\}$.
   X is said to be **closed** wrt K iff $X = CN_K(X)$. X is said to be **inconsistent** wrt K iff its closure $CN_K(X)$ is contradictory.
   X is **consistent** wrt K iff it is not inconsistent wrt K.
   K is said to be **consistent** iff its base of evidence BE_K is consistent wrt K.
4. Attack relations are defined as in Definition 5.
5. The basic postulates are defined as in Definition 6.
6. $\mathcal{R}$ and K are said to satisfy the self-contradiction property iff for each minimal inconsistent set $X \subseteq \mathcal{L}_{dom}$, for each $x \in X$, it holds:
   - $X \vdash_{RS} \neg x$ if $x \notin \mathcal{A}$
   - $X \vdash_{RS} \overline{x}$ if $x \in \mathcal{A}$

Notation 6.

- We say an argument A *undermines* an argument B (at $[\alpha]$) iff $conl(A) = \overline{\alpha}$, $\alpha \in \mathcal{A}$ and $[\alpha]$ is a subargument of B.
- The notions of undercut and rebut for extended knowledge bases are defined as in Definition 7.
- An argument A is said to be *generated by* a set S of arguments iff all basic defeasible subarguments of A as well as all assumption-subarguments of A are subarguments of arguments in S.
- Given an extended rule-based system $(\mathcal{R}, \mathcal{A})$, the *class of all consistent extended knowledge bases* of the form $(\mathcal{R}, \mathcal{A}, BE)$ is denoted by $\mathcal{E}_{\mathcal{R},\mathcal{A}}$ or just simply $\mathcal{E}_{\mathcal{R}}$ if no misunderstanding is possible.
- The notion of *attack relation assignments* are defined for extended knowledge bases in the same way as for knowledge bases (see Definition 16) where $\mathcal{C}_{\mathcal{R}}$ is replaced by $\mathcal{E}_{\mathcal{R}}$.
- The notion of *weakening* is defined for extended knowledge bases in the same way as for knowledge bases (see Definition 11).

It is not difficult to see that in Example 14, B undermines A_3 while A_1, A_2 as well as A_2, A_3 rebut each other.

Notation 7 *(Adaptations for extended knowledge bases).* All regular properties (see Definitions 9, 12, 14, 17), except the properties of effective rebuts and strong subargument structure, can be directly adopted for extended knowledge bases. We give the revised definition of properties of effective rebuts and strong subargument structure below.

We give an example explaining the reason for a slight revision of the effective rebut property.

Example 15. Let $\mathcal{R}$ be an extended rule-based system consisting only of two defeasible rules

$$d_0 :\Rightarrow a, \qquad d_1 : not\_a \Rightarrow \neg a$$

and a preference $d_0 \prec d_1$ where not_a is an assumption s.t. $\overline{not\_a} = a$.

It is clear that argument $A_0 \equiv [d_0]$ undermines argument $A_1 \equiv [[not\_a], d_1]$. Therefore even though $d_0 \prec d_1$, A_0 still attacks A_1.

Definition 34 *(Effective rebut for extended knowledge base).* We say that attack relation att satisfies the *effective rebut property* for an extended knowledge base K iff for all arguments $A_0, A_1 \in AR_K$ such that

- each A_i, $i = 0, 1$, contains exactly one defeasible rule d_i (i.e. $dr(A_i) = \{d_i\}$), and

[25] Note that X_{dom} could contain assumptions.

- A_0 rebuts A_1, and
- A_0 does not undermine A_1,[26]

it holds that A_0 attacks A_1 wrt att iff $d_0 \not\prec d_1$.

Definition 35 *(Strong subargument structure for extended knowledge base).* Attack relation att of an extended knowledge base K is said to satisfy the *property of strong subargument structure* iff for all $A, B \in AR_K$, the following conditions hold:

1. A attacks B (wrt att) iff A attacks a basic defeasible subargument or an assumption-subargument of B (wrt att).
2. If A undercuts or undermines B then A attacks B wrt att.
3. A directly attacks B (wrt att) then A undercuts or undermines or rebuts B (at B).

It is not difficult to see that in Example 14, for any regular attack relation, A_1 attacks A_2 (from the effective rebut property) and B attacks A_3 (from the strong subargument property).

It is straightforward to see that both Lemma 2 and Theorem 1 hold for extended knowledge bases.

Lemma 21. *Let att be an attack relation for extended knowledge base K satisfying the property of strong subargument structure. Further let E be a complete extension of (AR_K, att).*

1. *E contains all arguments generated by E, and*
2. *att satisfies the postulates of closure and subargument closure.*

Proof. Similar to the proof of Lemma 2. $\square$

Theorem 11. *Let att, att' be attack relations for knowledge base K.*

1. *If $att \subseteq att'$ and att is inconsistency-resolving for K then att' is also inconsistency-resolving for K.*
2. *If att satisfies the strong subargument structure and inconsistency-resolving properties then att satisfies the postulate of consistency.*

Proof. Identical to the proof of Theorem 1. $\square$

For any non-empty set $\mathcal{S}$ of attack relation assignments of extended rule-based system $\mathcal{R}$, define $\bigcup \mathcal{S}$ by:

$$\forall K \in \mathcal{E}_{\mathcal{R}}: \qquad (\bigcup \mathcal{S})(K) = \bigcup \{atts(K) \mid atts \in \mathcal{S}\}$$

The notions of weakly regular and regular attack relations assignments are defined for extended knowledge bases in the same way as for knowledge bases (see Definition 18).

The set of all regular attack relation assignments of an extended rule-based system $\mathcal{R}$ is denoted by $RAE_{\mathcal{R}}$.

Lemma 22. *Let $\mathcal{R}$ be an extended rule-based system and P be a regular property. Further let $\mathcal{S}$ be a non-empty set of attack relation assignments wrt $\mathcal{R}$ satisfying P. Then $\bigcup \mathcal{S}$ also satisfies P.*

Proof. See Appendix F. $\square$

From Lemma 22, the following Theorem 12 holds obviously.

Theorem 12. *Suppose $RAE_{\mathcal{R}}$ is not empty. Then $(RAE_{\mathcal{R}}, \subseteq)$ is a complete upper-semilattice.*

The greatest element in $(RAE_{\mathcal{R}}, \subseteq)$ is referred to as the canonical attack relation assignment of $\mathcal{R}$ and denoted by $Ate_{\mathcal{R}} = \bigcup RAE_{\mathcal{R}}$. $\square$

For extended rule-based systems, we define normal attack relation assignments in the same way as before, namely, for any extended knowledge base $K \in \mathcal{E}_{\mathcal{R}}$ and any arguments $A, B \in AR_K$, $(A, B) \in atts_{enr}(K)$ if and only if A undercuts or undermines B or A normal-rebuts B where the definition of normal-rebuts is the same like in section 5.2.

[26] Note that because A_0 rebuts A_1, A_0 can not undercut A_1.

Theorem 13.

1. *For any extended rule-based system $\mathcal{R}$, the normal attack relation assignment $atts_{enr}$ is weakly regular.*
2. *Suppose the extended rule-based system $\mathcal{R}$ satisfies the self-contradiction property. Then the normal attack relation assignment $atts_{enr}$ is regular and the canonical assignment $Ate_{\mathcal{R}}$ exists and $atts_{enr} \subseteq Ate_{\mathcal{R}}$.*

Proof. See Appendix F. □

Notation 8. The notion of well-prioritized rule-based system (Definition 26) is adopted directly for extended rule-based systems.

Theorem 14. *Let $\mathcal{R}$ be a well-prioritized extended rule-based system satisfying the self-contradiction property. The canonical attack relation assignment $Ate_{\mathcal{R}}$ and the normal attack relation assignment $atts_{enr}$ coincide.*

Proof. See Appendix F. □

We show next that canonical attack relations and normal attack relations for extended knowledge bases are also equivalent for stable semantics.

Theorem 15. *Suppose an extended rule-based system $\mathcal{R}$ satisfies the property of self-contradiction. Then for each $K \in \mathcal{C}_{\mathcal{R}}$, each stable extension wrt $atts_{enr}(K)$ is also a stable extension wrt $Ate_{\mathcal{R}}(K)$ and vice versa.*

Proof. See Appendix F. □

It is not difficult to generalize the results on least-fixed point characterization of canonical attack relation assignment for extended knowledge bases. While the removal functions REMOVE as well as FER, FAM, FLO, FSA, FCI could be adopted directly for the extended case, Lemmas 12, 26, 25 as well as Theorem 6 need to be revised slightly by $Ate_{\mathcal{R}} = Wate_{\mathcal{R}} = Batts_{uu} \cup (Batts_{rb} \setminus lfp(REMOVE))$ where $Wate_{\mathcal{R}}$ is the supremum of weakly regular attack relation assignments for extended rule-based system, $Batts_{uu}(K) = \{(A, B) \mid A, B \in AR_K, A$ undercuts or undermines $B\}$ and $Batts_{rb}(K) = \{(A, B) \mid A, B \in AR_K, A$ rebuts $B\}$. The reason for the slight modification is illustrated in Example 15. We omit the detailed proofs to avoid an unnecessarily long and repetitive presentation.

10. Conclusion

Ii essence, one can say that a key purpose of introducing preferences between defeasible rules is to rule out undesired attacks [4,43,47,30,25,23,24]. We develop this idea further by introducing the principle of minimal-removal of attacks stating that the removed attacks should be kept to a minimum. This principle is captured declaratively by the canonical attack relation assignment that is the greatest element in the complete upper-semilattice of regular attack relations. We also provide a least-fixed point characterization of the canonical attack relation assignment. We further show that for well-prioritized rule-based systems, canonical attack relation assignment coincides with the normal attack relation assignments providing an efficient characterization of canonical attack relations. It is also worthwhile to note that our framework could easily and naturally extended for general underlining language with negation-as-failure assumptions.

Other well-known approaches to structured argumentation are deductive argumentation of Besnard&Hunter [11], defeasible logic programming of Garcia&Simari [36,5] and assumption-based argumentation [32]. As the question of how and by which principles and guidelines a user of structured argumentation selects an attack relation for her/his domain is a common and fundamental problem for all approaches to structured argumentation with preferences, and as the regular properties are defined at an abstraction level easily applicable to other approaches of structured argumentation[27] and as the results we obtained in this paper hold for general underlining languages that could also include assumptions, we have convincing reasons to believe that the key results of this paper also hold in other approaches though their formal development could be different. This expectation is confirmed for assumption-based argumentation and ASPIC+ by our study of extended knowledge bases.[28] Further, section 8.1 also provides an in depth discussion about the relationship between

[27] A quick look at the property of inconsistency-resolving reveals that this definition could be applied to any approach to argumentation as long as a notion of consistency is present in it. Further the effective rebut property representing a minimal interpretation of preference between defeasible rule as well as the attack monotonicity property representing the intuition that an argument based on facts (like strict ones) should be preferred to the ones based on defeasible knowledge should obviously also hold across all approaches though their formal development could be different in different approaches. We could basically say the same on the definition of the context-independence property.

[28] In a recent paper, Cyras&Toni [20] have introduced priorities between assumptions into assumption-based argumentation. It would be interesting to see how the two approaches are related.

our approach and the other line of research that is based on defining pre-order between arguments before resolving their conflicts.

Brewka and Eiter [15] have proposed two principles for the evaluation of semantics of prioritized default reasoning. These principles, referred to as BE-principles, have been adapted to our framework in [23] and showed that both of them are satisfied by the ordinary attack relation assignments. As regular attack relation assignments are ordinary, they hence are also satisfied by regular attack relation assignments.[29]

A more liberal notion of unrestricted rebut where a basic defeasible argument could attack a non-basic defeasible argument is studied in [18,17]. Intuitively an unrestricted rebut is a rebut against a set of defeasible rules without explicitly rebutting any individual rule in it. As the notion of unrestricted rebut leads to counter-intuitive semantics wrt complete or stable extensions [17], the semantics of unrestricted rebuts is based on grounded extension. In [17,18], it is proved that grounded semantics wrt attack relations generated by the Aspic+ argument preference relations recalled in Definitions 30, 31, satisfies the consistency postulate if the preference relation $\preceq$ between defeasible rules is either empty or a total preorder. We show shortly below that in general when the preference relation $\preceq$ is neither empty nor a total preorder, the grounded semantics of unrestricted rebuts wrt ASPIC+ ap-assignments that are based on the elitist principle fails to satisfy the consistency postulate.

Consider the knowledge base K in Example 13. From $\{d_2, d_3, d_4\} \lhd_E \{d_1\}$, $\{d_1, d_3, d_4\} \lhd_E \{d_2\}$, $\{d_1, d_2, d_4\} \lhd_E \{d_3\}$, $\{d_1, d_2, d_3\} \lhd_E \{d_4\}$, it follows that for $1 \leq i \leq 4$, A_i attacks B_i but B_i does not attack A_i according to the ASPIC+ argument preferences based on the elitist principle and unrestricted rebuts. Therefore both attack relations based on ASPIC+ elitist principle equal $\{(A_i, B_i) \mid 1 \leq i \leq 4\}$.

It is obvious that the grounded extension (that is also stable) is $\{A_1, \ldots, A_4\}$ whose set of conclusion $\{a_1, \ldots, a_4\}$ is obviously inconsistent. Hence the consistency postulate is violated.

In general, the requirement that preference relation $\preceq$ should be total preorder is rather strong if we consider that the purpose of introducing preferences between rules is to resolve conflicts among them. Hence there is nothing wrong when there is no preference between rules if there are no conflicts to be resolved. Further imposing artificial priorities between rules could lead to contradictory semantics. For an illustration, consider again the arguments in Fig. 1. Two of the possible total preorders that are consistent with the preference $d_1 \prec d_2$ are $d_1 \prec d_2 \prec d$ and $d \prec d_1 \prec d_2$. Let us consider each of them.

- $d \prec d_1 \prec d_2$. It is obvious that $N'_1 \lhd_D A_2$ and $N_1 \lhd_D A_2$. Therefore there is only one stable extension E that is also grounded consisting of $[S], [d], A_2, N_2, N'_2$.
- $d_1 \prec d_2 \prec d$. It is obvious that $N'_1 \ntriangleleft_D A_2$. Therefore N'_1 attacks A_2. Hence apart from the stable extension E, the set $\{A_1, N_1, N'_1\}$ is part of another stable extension. The grounded extension consists only of the arguments $[S]$ and $[d]$.

The above discussion raises several interesting questions: Which one of the two cases should be viewed as "natural and intuitive"? What are the criteria for picking the "right total preorders" from a partial preorder of priorities between defeasible rules? Nevertheless, it would be interesting to see how the notion of unrestricted rebut interacts with the regular properties.

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Appendix A. Recall proof of Lemma 1

Note that for a strict argument A over a set $X \subseteq \mathcal{L}_{dom}$, the *set of premises of* A, $Prem(A)$, is defined by $Prem(A) = \{\alpha \in X \mid [\alpha]$ is a subargument of A$\}$.

Lemma 23. *Let $\mathcal{R}$ be a knowledge base closed under contraposition or transposition and A be a strict argument wrt $K = (\mathcal{R}, X)$ with conclusion σ. Then for each $\alpha \in Prem(A)$, there is a strict argument B wrt $(\mathcal{R}, X \cup \{\neg\sigma\})$ with $Prem(B) \subseteq Prem(A) \cup \{\neg\sigma\}$ and conclusion $\neg\alpha$.*

Proof. If $\mathcal{R}$ is closed under contraposition, the lemma is obvious. We prove the lemma for the case of closure under transposition by induction on the structure of A.

Base Case: $A = [\alpha]$, $\alpha \in X$. Obvious.

Inductive Case: Suppose A is of the form $[A_1, \ldots, A_n \to \sigma]$ where $cnl(A_i) = \alpha_i$. Let $\alpha \in Prem(A)$. Without loss of generality, let $\alpha \in Prem(A_n)$. From the closure under transposition, the rule $\alpha_1, \ldots, \alpha_{n-1}, \neg\sigma \to \neg\alpha_n$ also belongs to RS. Let B be the argument $A_1, \ldots, A_{n-1}, \neg\sigma \to \neg\alpha_n$.

From the induction hypothesis, there is an argument T whose premises are in $Prem(A_n) \cup \{\neg\alpha_n\}$ and whose conclusion is $\neg\alpha$.

[29] The readers are referred to [23] for further discussion.

Let T' be the argument obtained from T by replacing each occurrence of premise $\neg\alpha_n$ by the argument B. It is clear that $Prem(T') \subseteq Prem(A) \cup \{\neg\sigma\}$ and $cnl(T') = \neg\alpha$. □

Lemma 1. *If $\mathcal{R}$ is closed under transposition or contraposition then $\mathcal{R}$ satisfies the self-contradiction property.*

Proof. Let X be a minimal inconsistent subset of $\mathcal{L}_{dom}$. Since X is inconsistent, there is a $\lambda \in \mathcal{L}_{dom}$ such that $X \vdash_{\mathcal{R}} \lambda$ and $X \vdash_{\mathcal{R}} \neg\lambda$.

- Let $\mathcal{R}$ be closed under contraposition. Let $x \in X$. It is clear $\{x, \lambda\} \vdash_{\mathcal{R}} \lambda$. Since $\mathcal{R}$ is closed under contraposition, it follows obviously $\{\lambda, \neg\lambda\} \vdash_{\mathcal{R}} \neg x$. Therefore $X \vdash_{\mathcal{R}} \neg x$.
- Let $\mathcal{R}$ be closed under transposition. There are two arguments A_0, A_1 with premises in X and conclusions $\lambda, \neg\lambda$ respectively. From the minimality of X, it holds: $X = Prem(A_0) \cup Prem(A_1)$. Let $x \in X$. Without loss of generality, suppose $x \in Prem(A_0)$. From the Lemma 23, it follows that there exists an argument B with conclusion $\neg x$ and $Prem(B) \subseteq Prem(A_0) \cup \{\neg\lambda\}$. Let A be the argument obtained by replacing each subargument of the form $[\neg\lambda]$ in B by argument A_1. It is clear that $Prem(A) \subseteq X$ and the conclusion of A is $\neg x$. □

Appendix B. Upper semilattice of regular attack relations

Lemma 4. *Let $\mathcal{A}$ be a non-empty set of attack relation assignments.*

1. *Suppose P is a regular property and every attack relation assignment $atts \in \mathcal{A}$ satisfies P. Then $\bigcup\mathcal{A}$ also satisfies P.*
2. *If the attack relations assignments in $\mathcal{A}$ are regular then $\bigcup\mathcal{A}$ is also regular.*
3. *If the attack relation assignments in $\mathcal{A}$ are weakly regular then $\bigcup\mathcal{A}$ is also weakly regular.*

Proof. As statements 2, 3 follow immediately from (1), we need only to prove statement (1). Let $atts_0 = \bigcup\mathcal{A}$.

- Suppose every attack relation assignment $atts \in \mathcal{A}$ satisfies the property of strong subargument structure. We show that $atts_0$ also satisfies the property of strong subargument structure. Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$.
  - It is clear that A attacks B (wrt $atts_0(K)$) iff A attacks B (wrt $atts(K)$) for some $atts \in \mathcal{A}$ iff A attacks a basic defeasible subargument B_0 of B (wrt $atts(K)$) iff A attacks a basic defeasible subargument B_0 of B (wrt $atts_0(K)$).
  - It is obvious that if A undercuts B then A attacks B wrt $atts(K)$ for any $atts \in \mathcal{A}$ and hence A attacks B wrt $atts_0(K)$.
  - It is obvious that if A directly attacks B (wrt $atts_0(K)$) then A directly attacks B (wrt some $atts(K)$, $atts \in \mathcal{A}$) and hence A undercuts B (at B) or rebuts B (at B).
- Suppose every attack relation assignment $atts \in \mathcal{A}$ satisfies the inconsistency-resolving property. From Theorem 1, it follows immediately that $atts_0$ also satisfies the inconsistency-resolving property.
- It is obvious that $atts_0$ satisfies the effective rebut property if each attack relation assignment in $\mathcal{A}$ satisfies this property.
- Suppose every attack relation assignment $atts \in \mathcal{A}$ satisfies the attack monotonicity property. We show that $atts_0$ also satisfies this property. Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B, C, D \in AR_K$ such that C is a weakening of A and D is a weakening of B.
  - Suppose $(A, B) \in atts_0(K)$. There exists thus $atts \in \mathcal{A}$ s.t. $(A, B) \in atts(K)$. Since $atts(K)$ satisfies the attack monotonicity property, it follows $(A, D) \in atts(K)$. Therefore $(A, D) \in atts_0(K)$.
  - Suppose $(C, B) \in atts_0(K)$. There exists thus $atts \in \mathcal{A}$ s.t. $(C, B) \in atts(K)$. Since $atts(K)$ satisfies the attack monotonicity property, it follows $(A, B) \in atts(K)$. Therefore $(A, B) \in atts_0(K)$.
- Suppose every attack relation assignment $atts \in \mathcal{A}$ satisfies the link-orientation property. We show that $atts_0$ also satisfies this property. Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B, C \in AR_K$ such that C is a weakening of B by $AS \subseteq AR_K$ (i.e. $C \in B \downarrow AS$) and A attacks C (wrt $atts_0(K)$) and A does not attack AS (wrt $atts_0(K)$). We show that A attacks B (wrt $atts_0(K)$).
  From A attacks C (wrt $atts_0(K)$), it follows that there exists $atts \in \mathcal{A}$ s.t. $(A, C) \in atts(K)$.
  Because A does not attack AS (wrt $atts_0(K)$), A does not attack AS (wrt $atts'(K)$ for any $atts' \in \mathcal{A}$). Therefore A does not attack AS (wrt $atts(K)$. Since $atts$ satisfies the link-orientation property, it follows that A attacks B (wrt $atts(K)$). Therefore A attacks B (wrt $atts_0(K)$).
- Suppose every attack relation assignment $atts \in \mathcal{A}$ satisfies the context-independence property. We show that $atts_0$ also satisfies this property. Let $K, K' \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K \cap AR_{K'}$.
  It is clear that $(A, B) \in atts_0(K)$ iff there exists $atts \in \mathcal{A}$ s.t. $(A, B) \in atts(K)$ iff $(A, B) \in atts(K')$ (since $atts$ satisfies the context-independence property) iff $(A, B) \in atts_0(K')$. □

Theorem 3.

1. *For any rule-based system $\mathcal{R}$, the normal attack relation assignment $atts_{nr}$ is weakly regular.*
2. *Suppose the rule-based system $\mathcal{R}$ satisfies the self-contradiction property. Then the normal attack relation assignment $atts_{nr}$ is regular and the canonical assignment $Att_{\mathcal{R}}$ exists and $atts_{nr} \subseteq Att_{\mathcal{R}}$.*

Proof.

1. Let $\mathcal{R}$ be a rule-based system and $K \in \mathcal{C}_{\mathcal{R}}$.
   It is straightforward to see that $atts_{nr}$ satisfies the property of context-independence. It is also obvious that $atts_{nr}(K)$ satisfies the properties of strong subargument structure, effective rebuts.
   We show that $atts_{nr}(K)$ satisfies the property of link-orientation.
   Let $A, B, C \in AR_K$ for a knowledge base K such that C is a weakening of B by $AS \subseteq AR_K$ (i.e. $C \in B \downarrow AS$) and A does not attack AS wrt $atts_{nr}(K)$ and $(A, C) \in atts_{nr}(K)$. There is a basic defeasible subargument C' of C such that either $cnl(A) = ab_{last(C')}$ or $cnl(A) = \neg cnl(C')$ and there is no $d \in ldr(A)$ s.t. $d \prec last(C')$. Since A does not attack AS wrt $atts_{nr}(K)$, the defeasible rule $last(C')$ does not occur in any argument belonging to AS. Hence $last(C')$ occurs in B. Therefore $(A, B) \in atts_{nr}(K)$.
   We next show that $att_{nr}(K)$ satisfies the property of attack monotonicity.
   Let $K \in \mathcal{K}$ and C attacks B wrt $atts_{nr}(K)$ and C is a weakening of A. It is not difficult to see that if C undercuts B then A also undercuts B. Suppose now that C rebuts B (at B') and there is no $d \in ldr(C)$ s.t. $d \prec last(B')$. From $ldr(A) \subseteq ldr(C)$ and cnl(C) = cnl(A), it follows obviously that A rebuts B (at B') and there is no $d \in ldr(A)$ s.t. $d \prec last(B')$. We have proved that A also attacks B wrt att_{nr}.
   Suppose A attacks B wrt $atts_{nr}(K)$ and D is a weakening of B. It is easy to see that there exists a basic defeasible subargument B' of B such that either $last(A) = ab_{last(B')}$ or A normal-rebuts B' (at B'). It is easy to see that there is a subargument D' of D such that D' is also a weakening of B'. Hence $last(D') = last(B')$ and D' is basic defeasible. It holds obviously that either $last(A) = ab_{last(D')}$ or A normal-rebuts D' (at D'). A thus attacks D wrt $atts_{nr}(K)$.
2. We only need to show that $atts_{nr}(K)$ satisfies the inconsistency-resolving property.
   Let $K \in \mathcal{C}_{\mathcal{R}}$. Let $S \subseteq AR_K$ s.t. S is inconsistent.
   Let $BE_0 = cnl(S)$. Since S is inconsistent, there are two strict arguments A, B of the knowledge base $(\mathcal{R}, BE_0)$ with contradictory conclusions. Let A' be weakening of A by replacing each subargument $[e]$, $e \in BE_0$, of A by an argument in S with conclusion e. B' is obtained by weakening B in the similar way. Let BE_1 be the set of premises of arguments A', B'. It is clear that $BE_1 \subseteq BE_K$.
   Let MDA, MDB be the sets of maximal basic defeasible subarguments of A', B' respectively.
   Since A', B' have contradictory conclusion, it follows immediately that the set $cnl(MDA \cup MDB) \cup BE_1$ is inconsistent.
   Let C be a minimal inconsistent subset of $cnl(MDA \cup MDA) \cup BE_1$. Because K is consistent and $BE_1 \subseteq BE_K$, $C \setminus BE_1 \neq \emptyset$.
   Let S_0 be a minimal subset of $MDA_0 \cup MDA_1$ such that $cnl(S_0) = C \setminus BE_1$. S_0 therefore is non-empty and finite. Because S_0 is minimal and rules do not contain non-domain atoms in their bodies, $cnl(S_0)$ does not contain any non-domain atom.
   Let $LD = \{last(X) \mid X \in S_0\}$. LD is hence finite and non-empty. From the transitivity of $\preceq$, it follows that $\prec$ is a strict partial order. Therefore $\prec$ is a strict partial order on LD. Thus there exists a minimal element wrt $\prec$ in LD. Let $A \in S_0$ s.t. last(A) is minimal (wrt $\prec$) in LD. Since $cnl(S_0)$ does not contain any non-domain atom, $hd(last(A)) \in \mathcal{L}_{dom}$. From the self-contradiction property, $C \vdash \neg hd(last(A))$. We could then construct an argument B such that B rebuts A (at A) and all maximal basic defeasible subarguments of B are arguments in S_0. Therefore $ldr(B) \subseteq LD$. Since last(A) is minimal (wrt $\prec$) in LD, there is no $d \in ldr(B)$ s.t. $d \prec last(A)$. Therefore B normal-rebuts A, i.e. B attacks A wrt $atts_{nr}(K)$ (what we need to prove). $\square$

Lemma 7.

1. *Batts satisfies all regular properties except the inconsistency-resolving and effective rebut properties.*
2. *If $\mathcal{R}$ satisfies the self-contradiction property then Batts satisfies the inconsistency-resolving property.*
3. *If $\mathcal{R}$ is basic then Batts satisfies the effective rebut property and hence is weakly regular and $Batts = Watt_{\mathcal{R}}$.*

Proof. Let $\mathcal{R} = (RS, RD, \preceq)$. Further let $\mathcal{R}' = (RS, RD, \emptyset)$. Let $atts'_{nr}$ be the normal attack relation assignment wrt $\mathcal{R}'$. It is clear that $atts'_{nr} = Batts$. From Theorem 3 (note that Batts satisfies the effective rebut property for $\mathcal{R}'$, not for $\mathcal{R}$), it is clear that assertions 1,2 hold. If $\mathcal{R}$ is basic, $\mathcal{R} = \mathcal{R}'$. Hence assertion 3 also holds. $\square$

Appendix C. Minimal removal semantics

Lemma 11.

1. *All removal functions FER, FAM, FLO, FCI, FSA are continuous, i.e. for each directed $\Delta \subseteq REMAS_{\mathcal{R}}$, for each $\mathcal{X} \in \{FER, FAM, FLO, FCI, FSA\}$, it holds:*

$$\mathcal{X}(\bigcup \Delta) = \bigcup \mathcal{X}(\Delta)$$

2. *REMOVE is continuous and*

$$lfp(REMOVE) = \bigcup_{i=1}^{\infty} REMOVE^i(\overline{\emptyset})$$

Proof. Assertion 2 follows directly from assertion 1. We prove assertion 1 below.

From the definition of FER, it is easy to see that $FER(\pi)$ is constant, i.e. for any removal assignments π, π', $FER(\pi) = FER(\pi')$. It is hence obvious that $FER(\bigcup \Delta) = \bigcup FER(\Delta)$.

Let $\mathcal{X} \in \{FAM, FLO, FCI, FSA\}$.

From Lemma 9, it is clear that $\mathcal{X}(\bigcup \Delta) \supseteq \bigcup \mathcal{X}(\Delta)$.

It remains to be shown that $\mathcal{X}(\bigcup \Delta) \subseteq \bigcup \mathcal{X}(\Delta)$.

1. Let $\mathcal{X} = FAM$ and $K \in \mathcal{C}_{\mathcal{R}}$, $(A, B) \in FAM(\bigcup \Delta)(K)$.
   Therefore
   – there is a weakening B' of B such that $(A, B') \in \bigcup_{\pi \in \Delta} \pi(K)$, or
   – there exists a strengthening X of A such that $(X, B) \in \bigcup_{\pi \in \Delta} \pi(K)$.
   Hence
   – there is a weakening B' of B and a $\pi \in \Delta$ such that $(A, B') \in \pi(K)$, or
   – there exists a strengthening X of A and a $\pi' \in \Delta$ such that $(X, B) \in \pi'(K)$.
   Thus
   – there is a $\pi \in \Delta$ such that $(A, B) \in FAM(\pi)(K)$, or
   – there exists a $\pi' \in \Delta$ such that $(A, B) \in FAM(\pi')(K)$.
   It is obvious $(A, B) \in \bigcup_{\pi \in \Delta} FAM(\pi)(K)$.
2. Let $\mathcal{X} = FLO$ and $K \in \mathcal{C}_{\mathcal{R}}$.

$$\begin{aligned} FLO(\bigcup \Delta)(K) = \{(A,B) \mid\ & A, B \in AR_K,\ \text{A rebuts } B \text{ and} \\ & \exists B_0 \in AR_K, AS \subseteq AR_K \text{ such that} \\ & B \in B_0 \downarrow AS,\ \text{and} \\ & \text{if A rebuts } B_0 \text{ then } (A, B_0) \in \bigcup_{\pi \in \Delta} \pi(K),\ \text{and} \\ & \forall X \in AS:\ \text{if A rebuts X then } (A, X) \in \bigcup_{\pi \in \Delta} \pi(K)\} \\ = \{(A,B) \mid\ & A, B \in AR_K,\ \text{A rebuts B and} \\ & \exists B_0 \in AR_K, AS \subseteq AR_K \text{ such that} \\ & B \in B_0 \downarrow AS,\ \text{and} \\ & \text{if A rebuts } B_0 \text{ then } \exists \pi_0 \in \Delta : (A, B_0) \in \pi_0(K),\ \text{and} \\ & \forall X \in AS:\ \text{if A rebuts X then } \exists \pi_X \in \Delta : (A, X) \in \pi_X(K)\} \end{aligned}$$

   Without loss of generality, we could assume that AS is finite. Further, due to the directedness of Δ, there exists $\pi \in \Delta$ s.t. $\pi_0 \subseteq \pi$ and $\forall X \in AS : \pi_X \subseteq \pi$. Therefore it holds:

$$\begin{aligned} = \{(A,B) \mid\ & A, B \in AR_K,\ \text{A rebuts B and} \\ & \exists B_0 \in AR_K, AS \subseteq AR_K, \exists \pi \in \Delta \text{ such that} \\ & B \in B_0 \downarrow AS,\ \text{and} \\ & \text{if A rebuts } B_0 \text{ then } (A, B_0) \in \pi(K),\ \text{and} \\ & \forall X \in AS:\ \text{if A rebuts X then } (A, X) \in \pi(K)\} \\ \subseteq\ & \bigcup_{\pi \in \Delta} FLO(\pi)(K). \end{aligned}$$

3. Let $\mathcal{X} = FCI$ and $K \in \mathcal{C}_{\mathcal{R}}$.

$$\begin{aligned} FCI(\bigcup \Delta)(K) &= \{(A,B) \mid \text{A rebuts B and } \exists K' \in \mathcal{C}_{\mathcal{R}} : (A,B) \in \bigcup_{\pi \in \Delta} \pi(K')\} \\ &= \{(A,B) \mid \text{A rebuts B and } \exists K' \in \mathcal{C}_{\mathcal{R}}, \exists \pi \in \Delta : (A,B) \in \pi(K')\} \\ &\subseteq \bigcup_{\pi \in \Delta} FCI(\pi)(K). \end{aligned}$$

4.

$$
\begin{aligned}
FSA(\bigcup\Delta)(K) = \{(A,B) \mid & \text{ A rebuts B and}\\
& \text{for each basic defeasible subargument X of B:}\\
& \text{if A rebuts X (at X) then } (A,X) \in \bigcup_{\pi\in\Delta} \pi(K)\}\\
= \{(A,B) \mid & \text{ A rebuts B and}\\
& \exists \pi \in \Delta \text{ s.t. for each basic defeasible subargument X of B:}\\
& \text{if A rebuts X (at X) then } (A,X) \in \pi(K)\}\\
& \text{(because B has only finitely many basic defeasible subarguments)}\\
\subseteq \bigcup_{\pi\in\Delta} & FSA(\pi)(K). \qquad \square
\end{aligned}
$$

Lemma 12. *$Watt_{\mathcal{R}} = Batts \setminus lfp(REMOVE)$, i.e. $Watt_{\mathcal{R}}(K) = Batts(K) \setminus lfp(REMOVE)(K)$ for each $K \in \mathcal{C}_{\mathcal{R}}$.*

Proof. The lemma follows directly from Lemmas 26, 25 below. $\square$

Before proving Lemmas 26, 25 below, we need another lemma.

From now until the end of this section, let

$$\pi_i = REMOVE^i(\overline{\emptyset}) \quad \text{and} \quad \pi = lfp(REMOVE)$$

Lemma 24. *For each i, for each $K \in \mathcal{C}_{\mathcal{R}}$, for all $A, B \in AR_K$ if $(A, B) \in \pi_i(K)$ then for each basic defeasible subargument X of B, if A rebuts X then $(A, X) \in \pi_i(K)$.*

Proof. By induction. It is obvious that the assertion holds for $i = 0$.

Suppose the assertion holds for i. We show that it holds for $i + 1$.

Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$ such that $(A, B) \in \pi_{i+1}(K)$. It follows immediately that A rebuts B. From $\pi_{i+1} = REMOVE(\pi_i)$, it follows $(A, B) \in REMOVE(\pi_i)$.

1. "$(A, B) \in FER(\pi_i)(K)$". Let X be a basic defeasible subargument of B such that A rebuts X. It is clear that $dr(B) = dr(X)$. Therefore from the definition of FER, $(A, X) \in FER(\pi_i)(K)$.
2. "$(A, B) \in FAM(\pi_i)(K)$".
   Therefore
   - there is a weakening B' of B such that $(A, B') \in \pi_i(K)$, or
   - there exists a strengthening Z of A such that $(Z, B) \in \pi_i(K)$.

   We consider each case in turn.
   - Suppose there is a weakening B' of B such that $(A, B') \in \pi_i(K)$.
     From the induction hypothesis, for each basic defeasible subargument Y of B', if A rebuts Y then $(A, Y) \in \pi_i(K)$.
     Let X be a basic defeasible subargument of B such that A rebuts X. Therefore there is a basic defeasible subargument Y of B' that is a weakening of X. Therefore A rebuts Y. From $(A, Y) \in \pi_i(K)$, it follows that $(A, X) \in FAM(\pi_i)(K)$. Thus $(A, X) \in \pi_{i+1}(K)$.
   - Suppose there exists a strengthening Z of A such that $(Z, B) \in \pi_i(K)$. From the induction hypothesis, for all basic defeasible subargument X of B, if Z rebuts X then $(Z, X) \in \pi_i(K)$. It is clear that Z rebuts some argument iff A rebuts the same argument. Since Z is a strengthening of A, it follows that for each basic defeasible subargument X of B, if A rebuts X then $(Z, X) \in \pi_i(K)$. Since Z is a strengthening of A, from the definition of FAM, it follows directly $(A, X) \in FAM(\pi_i)(K) \subseteq \pi_{i+1}(K)$.
3. "$(A, B) \in FLO(\pi_i)(K)$". It holds that
   A rebuts B and
   $\exists B_0 \in AR_K, AS \subseteq AR_K$ such that
   $B \in B_0 \downarrow AS$, and if A rebuts B_0 then $(A, B_0) \in \pi_i(K)$, and
   $\forall Z \in AS$: if A rebuts Z then $(A, Z) \in \pi_i(K)$.
   Let X be a basic defeasible subargument of B such that A rebuts X. There are two cases:
   - X is a subargument of some $Z \in AS$, or
   - X is not a subargument of any $Z \in AS$.

   We consider each case in turn.

– "X is a subargument of some $Z \in AS$".
Therefore A rebuts Z. Hence $(A, Z) \in \pi_i(K)$. From the induction hypothesis, $(A, X) \in \pi_i(K)$.
– "X is not a subargument of any $Z \in AS$".
Therefore there exists a subargument X_0 of B_0 such that $X \in X_0 \downarrow AS$. Since X is basic defeasible, X_0 is basic defeasible.
There are two cases:
- A does not rebut X_0. Therefore the condition "if A rebuts X_0 then $(A, X_0) \in \pi_i(K)$" holds. Therefore, it holds that $X \in X_0 \downarrow AS$, and if A rebuts X_0 then $(A, X_0) \in \pi_i(K)$, and
$\forall Z \in AS:$ if A rebuts Z then $(A, Z) \in \pi_i(K)$.
Therefore, $(A, X) \in FLO(\pi_i)(K) \subseteq \pi_{i+1}(K)$.
- Suppose A rebuts X_0. Therefore A rebuts B_0. Hence $(A, B_0) \in \pi_i(K)$. Since X_0 is a basic defeasible subargument of X, it follows from the induction hypothesis that $(A, X_0) \in \pi_i(K)$. Since $X \in X_0 \downarrow AS$, it follows from the definition of FLO that $(A, X) \in FLO(\pi_i)(K) \subseteq \pi_{i+1}(K)$.

4. "$(A, B) \in FCI(\pi_i)(K)$". Therefore A rebuts B and $\exists K' \in \mathcal{C}_\mathcal{R} : (A, B) \in \pi_i(K')$.
Let X be a basic defeasible subargument of B such that A rebuts X. From the induction hypothesis, $(A, X) \in \pi_i(K')$. Therefore $(A, X) \in FCI(\pi_i)(K)$.
5. "$(A, B) \in FSA(\pi_i)(K)$".
Therefore A rebuts B and for each basic defeasible subargument Z of B: if A rebuts Z (at Z) then $(A, Z) \in \pi_i(K)$.
Let X be a basic defeasible subargument of B s.t. A rebuts X. Thus for each basic defeasible subargument Y of X: if A rebuts Y (at Y) then $(A, Y) \in \pi_i(K)$. Hence $(A, X) \in FSA(\pi_i)(K)$. □

Lemma 25. *For each weakly regular attack relation $atts \in WRAA_\mathcal{R}$, it holds:*

$$atts \subseteq Batts \setminus lfp(REMOVE)$$

Proof. Let atts be a weakly regular attack relation assignment for $\mathcal{R}$. It is clear that $atts \subseteq Batts \setminus lfp(REMOVE)$ iff $atts \cap lfp(REMOVE) = \overline{\emptyset}$.

We show below that for each $K \in \mathcal{C}_\mathcal{R}$, $atts(K) \cap \pi(K) = \emptyset$.
We show by induction that for each $K \in \mathcal{C}_\mathcal{R}$, $atts(K) \cap \pi_i(K) = \emptyset$ holds.
As $\pi_0 = \overline{\emptyset}$, it is clear that $atts(K) \cap \pi_0(K) = \emptyset$.
Suppose $atts(K) \cap \pi_i(K) = \emptyset$ holds. We want to show that $atts(K) \cap \pi_{i+1}(K) = \emptyset$ also holds.
Suppose $\exists (X, Y) \in atts(K) \cap \pi_{i+1}(K)$. From $\pi_{i+1} = REMOVE(\pi_i)$, there are five cases to consider.

1. Suppose $(X, Y) \in FER(\pi_i)(K)$. Therefore each of X, Y contains exactly one defeasible rule. Let the defeasible rules in X, Y be respectively d, d'. Since X rebuts Y, it holds that $d \prec d'$.
Since $atts$ satisfies the effective rebut property, it follows that $(X, Y) \notin atts(K)$. Contradiction. Hence $(X, Y) \notin FER(\pi_i)$.
2. Suppose $(X, Y) \in FAM(\pi_i)(K)$.
Therefore
– there is a weakening Y' of Y such that $(X, Y') \in \pi_i(K)$, or
– there exists a strengthening Z of X such that $(Z, Y) \in \pi_i(K)$.
From the induction hypothesis that $atts(K) \cap \pi_i(K) = \emptyset$, it follows that
– there is a weakening Y' of Y such that $(X, Y') \notin atts(K)$, or
– there exists a strengthening Z of X such that $(Z, Y) \notin atts(K)$.
Since $atts$ satisfies the attack monotonicity (and hence also a contrapositive reading of it), it holds that $(X, Y) \notin atts(K)$. Contradiction. Hence $(X, Y) \notin FAM(\pi_i)$.
3. Suppose $(X, Y) \in FLO(\pi_i)(K)$.
It follows that

$$\begin{aligned}
&\text{X rebuts Y and}\\
&\exists Y_0 \in AR_K, AS \subseteq AR_K \text{ such that}\\
&Y \in Y_0 \downarrow AS, \text{ and}\\
&\text{if X rebuts } Y_0 \text{ then } (X, Y_0) \in \pi_i(K), \text{ and}\\
&\forall Z \in AS : \text{if X rebuts Z then } (X, Z) \in \pi_i(K)
\end{aligned}$$

From the induction hypothesis that $atts(K) \cap \pi_i(K) = \emptyset$, it follows that

$$\begin{aligned}
&\text{X rebuts Y and}\\
&\exists Y_0 \in AR_K, AS \subseteq AR_K \text{ such that}\\
&Y \in Y_0 \downarrow AS, \text{ and}
\end{aligned}$$

$$\text{if X rebuts } Y_0 \text{ then } (X, Y_0) \notin atts(K), \text{ and}$$

$$\forall Z \in AS: \text{ if X rebuts Z then } (X, Z) \notin atts(K)$$

It follows that $(X, Y_0) \notin atts(K)$ and $\forall Z \in AS : (X, Z) \notin atts(K)$.
Since *atts* satisfies the link-orientation property (and hence also a contrapositive reading of it), it holds that $(X, Y) \notin atts(K)$. Contradiction. Hence $(X, Y) \notin FLO(\pi_i)$.

4. Suppose $(X, Y) \in FCI(\pi_i)(K)$.
Therefore X rebuts Y and $\exists K' \in \mathcal{C}_{\mathcal{R}}, \exists i : (X, Y) \in \pi_i(K')$.
From the induction hypothesis, it holds that X rebuts Y and $\exists K' \in \mathcal{C}_{\mathcal{R}} : X, Y \in AR_{K'}$ and $(X, Y) \notin atts(K')$.
Since *atts* satisfies the context-independence property (and hence also a contrapositive reading of it), it holds that $(X, Y) \notin atts(K)$. Contradiction. Hence $(X, Y) \notin FCI(\pi_i)$.
5. Suppose $(X, Y) \in FSA(\pi_i)(K)$.
Therefore X rebuts Y and for each basic defeasible subargument Z of Y: if X rebuts Z (at Z) then $(X, Z) \in \pi_i(K)$.
From the induction hypothesis that $atts(K) \cap \pi_i(K) = \emptyset$, it follows that
(*) X rebuts Y and for each basic defeasible subargument Z of Y: if X rebuts Z (at Z) then $(X, Z) \notin atts(K)$.
Since $(X, Y) \in atts(K)$ and *atts* satisfies the strong subargument structure property, X directly attacks (wrt atts(K)) some basic defeasible subargument C of Y. Therefore X rebuts C (at C). Hence $(X, C) \in atts(K)$. Contradiction to assertion (*).
Hence $(X, Y) \notin FSA(\pi_i)$.

Therefore $(X, Y) \notin \pi_{i+1}(K)$. Contradiction. Therefore there exists no $(X, Y) \in atts(K) \cap \pi_{i+1}(K)$. □

Lemma 26. *$Batts \setminus lfp(REMOVE)$ is weakly regular.*

Proof. Let $watt = Batts \setminus lfp(REMOVE)$.
It is not difficult to see that for each $K \in \mathcal{C}_{\mathcal{R}}$, for all arguments A, B, if A undercuts B then $(A, B) \in watt(K)$.

1. Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A_0, A_1 \in AR_K$ such that each A_i, $i = 0, 1$, contains exactly one defeasible rule d_i (i.e. $dr(A_i) = \{d_i\}$), and A_0 rebuts A_1.
Let $d_0 \not\prec d_1$. Therefore A_0 normal-rebuts A_1. From Theorem 3 and Lemma 25, it follows $atts_{nr} \subseteq watt$. Therefore $(A_0, A_1) \in watt(K)$.
Let $(A_0, A_1) \in watt(K)$. We show that $d_0 \not\prec d_1$. Suppose $d_0 \prec d_1$. Therefore from the definition of the removal function FER, it follows that $(A_0, A_1) \in FER(lfp(REMOVE))(K')$ for any $K' \in \mathcal{C}_{\mathcal{R}}$. Hence $(A_0, A_1) \in lfp(REMOVE)(K')$ for any $K' \in \mathcal{C}_{\mathcal{R}}$. Thus $(A_0, A_1) \notin watt(K)$. Contradiction.
We have proved that watt satisfies the effective rebut property.
2. Suppose watt does not satisfy the attack monotonicity property. Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, A', B, B' \in AR_K$ such that A' is a weakening of A and B' is a weakening of B and
(a) $(A, B) \in watt(K)$ and $(A, B') \notin watt(K)$, or
(b) $(A', B) \in watt(K)$ and $(A, B) \notin watt(K)$.
We consider each case in turn.
   – Suppose $(A, B) \in watt(K)$ and $(A, B') \notin watt(K)$. Since A rebuts B and B' is a weakening of B, it follows that A also rebuts B'. From $(A, B') \notin watt(K)$, it follows $(A, B') \in \pi(K)$. Hence $(A, B') \in \pi_i(K)$ for some i. From the definition of REMOVE and FAM, $(A, B) \in FAM(\pi_i)(K) \subseteq \pi_{i+1}(K) \subseteq \pi(K)$. Thus $(A, B) \notin watt(K)$. Contradiction.
   – $(A', B) \in watt(K)$ and $(A, B) \notin watt(K)$. Since A rebuts B and A' is a weakening of A, it follows that A' also rebuts B. From $(A, B) \notin watt(K)$, it follows $(A, B) \in \pi(K)$. Hence $(A, B) \in \pi_i(K)$ for some i. From the definition of REMOVE and FAM, $(A', B) \in FAM(\pi_i)(K) \subseteq \pi_{i+1}(K) \subseteq \pi(K)$. Thus $(A', B) \notin watt(K)$. Contradiction.
3. Suppose watt does not satisfy the link-orientation property. Therefore there exists $K \in \mathcal{C}_{\mathcal{R}}$ and there are arguments $A, B, C \in AR_K$ such that C is a weakening of B by $AS \subseteq AR_K$ (i.e. $C \in B \downarrow AS$) and A attacks C (wrt watt(K)) and A does not attack AS (wrt watt(K)) and A does not attack B (wrt watt(K)).
It is obvious that A does not undercut C (otherwise, A would undercut B or some in AS, and hence attacks B or some in AS). Therefore A rebuts C. Since A does not attack AS (wrt watt(K)), it follows obviously that if A rebuts any argument $X \in AS$ then $(A, X) \notin watt(K)$. From $watt = Batts \setminus \pi$, it holds that if A rebuts any argument $X \in AS$ then $(A, X) \in \pi(K)$. As A does not attack B (wrt watt(K)), it follows that if A rebuts B then $(A, B) \in \pi(K)$. Therefore $(A, C) \in FLO(\pi)(K) \subseteq \pi(K)$. From $watt = Batts \setminus \pi$, it follows that $(A, C) \notin watt(K)$. Contradiction. Hence we have proved that watt satisfies the link-orientation property.
4. Suppose watt does not satisfy the context-independence property. Therefore there are two knowledge bases $K, K' \in \mathcal{C}_{\mathcal{R}}$ and there are two arguments A, B from $AR_K \cap AR_{K'}$ such that $(A, B) \in watt(K)$ and $(A, B) \notin watt(K')$. It is obvious that A does not undercut B. Therefore A rebuts B. From $(A, B) \notin watt(K')$ and $watt = Batts \setminus \pi$, it follows that $(A, B) \in \pi(K')$. Hence $(A, B) \in FCI(\pi)(K)$. Thus $(A, B) \notin watt(K)$. Contradiction.
We have proved that watt satisfies the context-independence property.
5. We show that watt satisfies the strong subargument property by showing that each of the three assertions in Definition 8 holds.

(a) It is obvious that if A undercuts B then A attacks B wrt $watt(K)$.
(b) Suppose there are $A, B \in AR_K$ s.t. A directly attacks B (wrt $watt(K)$).
Since $watt \subseteq Batts$, it follows that A undercuts or rebuts B.
Since A directly attacks B (wrt $watt(K)$), if A undercuts B then it is clear that A undercuts B (at B).
Suppose now that A rebuts B. We show that A rebuts B (at B). Suppose A does not rebut B (at B).
Since A directly attacks B wrt watt(K) it follows that for each basic defeasible subargument X of B, if A rebuts X (at X) then $X \neq B$ and $(A, X) \notin watt(K)$. Hence for each basic defeasible subargument X of B, if A rebuts X (at X) then $(A, X) \in \pi(K)$. Therefore $(A, B) \in FSA(\pi)(K)$. Hence $(A, B) \in \pi(K)$. Thus $(A, B) \notin watt(K)$. Contradiction. This case can not happen.
(c) We show that for each $K \in \mathcal{C}_{\mathcal{R}}$, for all $A, B \in AR_K$, it holds that A attacks B (wrt $watt(K)$) iff A attacks a basic defeasible subargument of B (wrt $watt(K)$).
"$\Rightarrow$" We show that for each $K \in \mathcal{C}_{\mathcal{R}}$, for all $A, B \in AR_K$, if A attacks B (wrt $watt(K)$) then A attacks a basic defeasible subargument of B (wrt $watt(K)$).
Suppose on the contrary there is $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$ such that A attacks B (wrt $watt(K)$) and A does not attack any basic defeasible subargument of B (wrt $watt(K)$).
Therefore A rebuts B. Since A does not attack any basic defeasible subargument of B (wrt $watt(K)$), and $watt = Batts \setminus \pi$, it holds that for any basic defeasible subargument Z of B, if A rebuts Z then $(A, Z) \in \pi(K)$. Therefore $(A, B) \in FSA(\pi)(K) \subseteq \pi(K)$. Hence $(A, B) \notin watt(K)$. Contradiction. This case can not happen.
"$\Leftarrow$" We show that for each $K \in \mathcal{C}_{\mathcal{R}}$, for all $A, B \in AR_K$, if A attacks a basic defeasible subargument of B (wrt $watt(K)$) then A attacks B (wrt $watt(K)$).
Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$ s.t. A attacks a basic defeasible subargument X of B (wrt $watt(K)$). We show that A attacks B (wrt $watt(K)$).
If A undercuts a basic defeasible subargument of B then A undercuts B. There is nothing to prove.
Let A rebut a basic defeasible subargument of B. Therefore A rebuts B. Suppose A does not attack B (wrt $watt(K)$). Therefore $(A, B) \in \pi(K)$. From Lemma 24, it follows that $(A, X) \in \pi(K)$. Contradiction. We have proved that A attacks B wrt $watt(K)$. □

Appendix D. Canonical and normal attack relation assignment

Lemma 13. *Let atts be a regular attack relation assignment for $\mathcal{R}$. Further let $K \in \mathcal{C}_{\mathcal{R}}$, $A, B \in AR_K$ and $d \in ldr(A)$ such that the following properties hold:*

- *A attacks B (wrt atts(K)).*
- *A rebuts B (at B) and A does not rebut any proper subargument of B.*
- *Both $str(B, last(B))$ and $str(A, d)$ belong to AR_K.*

The following conditions hold:

1. $str(A, d)$ attacks $str(B, last(B))$ (wrt atts(K)).
2. $d \not\prec last(B)$.

Proof.

1. From the attack monotonicity, $str(A, d)$ attacks B. Since A does not rebut any proper subargument of B and the conclusion of A (and hence also of $str(A, d)$) belongs to $\mathcal{L}_{dom}$, $str(A, d)$ does not attack any proper subargument of B. Therefore from the link-orientation property, $str(A, d)$ attacks $str(B, last(B))$.
2. Since atts is regular and $str(A, d)$ attacks $str(B, last(B))$, the effective rebut property directly implies $d \not\prec last(B)$. □

Lemma 14. *Suppose $\mathcal{R}$ be a well-prioritized rule-based system. Further let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$ and $d \in ldr(A)$ such that A rebuts B (at B) and $d \prec last(B)$. Then $(A, B) \notin Att_{\mathcal{R}}(K)$.*

Proof. Suppose $(A, B) \in Att_{\mathcal{R}}(K)$. Let BE_0 be the set of evidences from BE_K on which arguments A,B are based, i.e. $BE_0 = \{e \in BE_K \mid [e]$ is a subargument of A or B$\}$.

Let $A_0 = str(A, d)$ and $B_0 = str(B, last(B))$. Let BE_1 be the set of evidences on which the arguments A_0, B_0 are based.

Since $d \prec last(B)$, it follows from the well-prioritizedness of $\mathcal{R}$ that $\Delta(bd(last(B))) \cup \Delta(\neg hd(last(B)))$ is consistent. Therefore $hd(last(B)) \notin \Delta(bd(last(B)))$. Since $cnl(A) = \neg hd(last(B))$, it follows that A does not rebut any proper subargument of B.

From $bd(last(B)) \subseteq \Delta(bd(last(B)))$, it follows $BE' = BE_0 \cup BE_1 \subseteq \Delta(cnl(A)) \cup \Delta(bd(last(B)) = \Delta(\neg hd(last(B))) \cup \Delta(bd(last(B))$.

From the well-prioritizedness of $\mathcal{R}$, BE' is consistent. Therefore $K' = (\mathcal{R}, BE') \in \mathcal{C}_{\mathcal{R}}$ and the arguments A, B, A_0, B_0 belong to $AR_{K'}$. From the context-independence property, it follows that A attacks B wrt $Att_{\mathcal{R}}(K')$ by rebut at B.

We have proved that A attacks B wrt $Att_{\mathcal{R}}(K')$, A rebuts B (at B), and A does not rebut any proper subargument of B and A, B, A_0, B_0 belong to $AR_{K'}$. From assertion 2 of Lemma 13, it follows that $d \not\prec last(B)$. Contradiction. We have proved that $(A, B) \notin Att_{\mathcal{R}}(K)$ holds. $\square$

Lemma 18. *Let $\mathcal{R}$ be a rule-based system satisfying the self-contradiction property and atts be a regular attack relation assignment of $\mathcal{R}$. Further let $K \in \mathcal{C}_{\mathcal{R}}$. Each stable extension of $(AR_K, atts(K))$ is also a stable extension of $(AR_K, atts_{nr}(K))$.*

Proof. Let $K = (\mathcal{R}, BE)$ and E be a stable extension of $(AR_K, atts(K))$.

- From Theorem 3, Lemma 2 and Lemma 16, it is clear that E is conflict-free wrt $atts_{nr}(K)$.
- We show that E attacks (wrt $atts_{nr}(K)$) each argument not belonging to E. Let $B \in AR_K \setminus E$. Without loss of generality, we can assume that all proper subarguments of B belong to E (otherwise just pick a subargument of B satisfying this property). From Lemma 2, it follows immediately that B is basic defeasible.
  Since each proper subargument of B belongs to E, no proper subargument of B is rebutted by any argument in E.
  Since E is stable wrt $atts(K)$, there is $A \in E$ s.t. A attacks B wrt $atts(K)$.
  If A undercuts B then A attacks B (wrt $atts_{nr}(K)$).
  Suppose that A rebuts B. Therefore A rebuts B (at B) and A does not rebut any proper subargument of B.
  There are two cases:
  - A is strict. It is obvious that A normal-rebuts B.
  - A is defeasible. Let S be the set consisting of all evidences on which all arguments of the form $str(A, d)$, $d \in ldr(A)$ and $str(B, last(B))$ are based. It is clear that $S \subseteq cnl(E)$. Let $BE' = BE \cup S$. It is clear that $K' = (\mathcal{R}, BE') \in \mathcal{C}_{\mathcal{R}}$. Therefore $str(A, d)$, $d \in ldr(A)$ and $str(B, last(B))$ all belong to $AR_{K'}$.
    Since atts is regular and hence satisfies the context-independence property, A attacks B wrt $atts(K')$. We have proved that A rebuts B (at B) and A does not rebut any proper subargument of B. Further for each $d \in ldr(A)$, both $str(A, d)$ and $str(B, last(B))$ all belong to $AR_{K'}$.
    From Lemma 13, it follows that there is no $d \in ldr(A)$ s.t. $d \prec last(B)$. Therefore A normal-rebuts B, i.e. A attacks B wrt normal attack relation $atts_{nr}(K)$.

We have proved that E is stable wrt $atts_{nr}(K)$. $\square$

Appendix E. Discussion

Lemma 20. *Let $\mathcal{P}$ be a non-empty set of regular apr-assignments for $\mathcal{R}$. Then $\sqcap\mathcal{P}$ is regular.*

Proof. We show the equation $atts_{\sqcap\mathcal{P}} = \bigcup\{atts_\Gamma \mid \Gamma \in \mathcal{P}\}$. The regularity of $\sqcap\mathcal{P}$ follows from Lemma 4.

1. We show $atts_{\sqcap\mathcal{P}}(K) \subseteq \bigcup\{atts_\Gamma \mid \Gamma \in \mathcal{P}\}$.
   Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$ such that $(A, B) \in atts_{\sqcap\mathcal{P}}(K)$.
   If A undercuts B then it is obvious that $(A, B) \in atts_\Gamma(K)$ for each $\Gamma \in \mathcal{P}$. Hence $(A, B) \in \bigcup\{atts_\Gamma \mid \Gamma \in \mathcal{P}\}(K)$.
   Suppose A rebuts B (at B') and $A \not\sqsubset_{\sqcap\mathcal{P},K} B'$.
   From $\sqsubset_{\sqcap\mathcal{P},K} = \bigcap\{\sqsubset_{\Gamma,K} \mid \Gamma \in \mathcal{P}\}$, it follows that $A \not\sqsubset_{\sqcap\mathcal{P},K} B'$ implies that $A \not\sqsubset_{\Gamma,K} B'$ for some $\Gamma \in \mathcal{P}$.
   From A rebuts B (at B') and $A \not\sqsubset_{\Gamma,K} B'$ for $\Gamma \in \mathcal{P}$, it follows that $(A, B) \in atts_\Gamma(K)$. Hence $(A, B) \in \bigcup\{atts_\Gamma \mid \Gamma \in \mathcal{P}\}(K)$.
2. We show $\bigcup\{atts_\Gamma \mid \Gamma \in \mathcal{P}\} \subseteq atts_{\sqcap\mathcal{P}}(K)$.
   Let $K \in \mathcal{C}_{\mathcal{R}}$ and $A, B \in AR_K$ such that $(A, B) \in atts_\Gamma(K)$ for some $\Gamma \in \mathcal{P}$.
   If A undercuts B then it is obvious that $(A, B) \in atts_{\sqcap\mathcal{P}}(K)$.
   Suppose A rebuts B (at B') and $A \not\sqsubset_{\Gamma,K} B'$ for $\Gamma \in \mathcal{P}$. Hence $(A, B') \notin \bigcap\{\sqsubset_{\Gamma,K} \mid \Gamma \in \mathcal{P}\}$ implying that $A \not\sqsubset_{\sqcap\mathcal{P},K} B'$. Therefore $(A, B) \in atts_{\sqcap\mathcal{P}}(K)$. $\square$

Theorem 9.

1. *If $AP_{\mathcal{R}}$ is non-empty then $(AP_{\mathcal{R}}, \ll, \sqcap)$ forms a lower semilattice with $CA_{\mathcal{R}} = \sqcap AP_{\mathcal{R}}$ being the least regular ap-assignment for $\mathcal{R}$ and is referred to as the* **canonical ap-assignment**.
2. *If the rule-based system $\mathcal{R}$ satisfies the self-contradiction property then the set of regular ap-assignments $AP_{\mathcal{R}}$ is not empty and $atts_{CA_{\mathcal{R}}} = Att_{\mathcal{R}}$.*

Proof.

1. The first assertion follows immediately from Lemma 20.
2. Define an ap-assignment Γ as follows: Let $K \in \mathcal{R}$ and $A, B \in AR_K$. $A \sqsubseteq_{\Gamma,K} B$ iff A rebuts B (at B) and $(A, B) \notin Att_{\mathcal{R}}(K)$.
   We show that $\sqsubseteq_{\Gamma,K} = \sqsubset_{\Gamma,K}$.

Suppose on the contrary there are $X \sqsubseteq_{\Gamma,K} Y$ and $Y \sqsubseteq_{\Gamma,K} X$. Therefore X rebuts Y (at Y) and Y rebuts X (at X) and $(X,Y),(Y,X) \notin Att_{\mathcal{R}}(K)$. Since $att_{nr}(K) \subseteq Att_{\mathcal{R}}(K)$ (Theorem 3), it follows $(X,Y),(Y,X) \notin att_{nr}(K)$. Hence $last(X) \prec last(Y)$ and $last(Y) \prec last(X)$. Contradiction. We have proved that $\sqsubseteq_{\Gamma,K} = \sqsubset_{\Gamma,K}$.
We show $atts_{\Gamma}(K) = Att_{\mathcal{R}}(K)$.
Let $(X,Y) \in atts_{\Gamma}(K)$ and X does not undercut Y. Therefore X rebuts Y (at Y') and $X \not\sqsubset_{\Gamma,K} Y'$. Therefore $(X,Y') \in Att_{\mathcal{R}}(K)$. Since $Att_{\mathcal{R}}$ is regular, $(X,Y) \in Att_{\mathcal{R}}(K)$.
Let $(X,Y) \in Att_{\mathcal{R}}(K)$ and X does not undercut Y. Therefore there exists a basic defeasible argument Y' of Y such that X directly attacks Y' (wrt $Att_{\mathcal{R}}(K)$). Therefore X rebuts Y' (at Y') and $(X,Y') \in Att_{\mathcal{R}}(K)$. Hence $X \not\sqsubset_{\Gamma,K} Y'$. Therefore $(X,Y) \in atts_{\Gamma}(K)$.
Therefore $atts_{\Gamma} = Att_{\mathcal{R}}$. Therefore Γ is regular.
We show that Γ is the least regular ap-assignment.
Let $\Delta \in AP_{\mathcal{R}}$. We show that $\Gamma \ll \Delta$ holds.
Let $A \sqsubset_{\Gamma,K} B$. From the definition of Γ, it follows A rebuts B (at B) and $(A,B) \notin Att_{\mathcal{R}}(K)$. From $atts_{\Delta} \subseteq Att_{\mathcal{R}}$, it follows $(A,B) \notin atts_{\Delta}(K)$ implying that $A \sqsubset_{\Delta,K} B$. Since $\sqsubset_{\Gamma,K} = \sqsubseteq_{\Gamma,K}$, it follows $\Gamma \ll \Delta$.
As $CA_{\mathcal{R}}$ is the least regular ap-assignment, it holds that $\Gamma = CA_{\mathcal{R}}$. From $atts_{\Gamma} = Att_{\mathcal{R}}$, it follows that $atts_{CA_{\mathcal{R}}} = Att_{\mathcal{R}}$. □

Theorem 10. *The credulous cumulativity property is satisfied by all regular attack relation assignments.*

Proof. Let $\mathcal{R}$ be a rule-based system, $atts$ be a regular attack relation assignment of $\mathcal{R}$, $K \in \mathcal{C}_{\mathcal{R}}$ and E be a stable extension of $(AR_K, atts(K))$, $S = cnl(E)$ and $\Omega \subseteq S \cap \mathcal{L}_{dom}$ be a finite set of domain sentences. Further let $K' = K + \Omega$ and $E' = \{X \in AR_{K'} \mid \exists X' \in E, AS \subseteq E \text{ s.t. } cnl(AS) \subseteq \Omega \text{ and } X' \in X \downarrow AS\}$.[30] We show that E' is a stable extension of $(AR_{K'}, att(K'))$ by showing that it is conflict-free and attacks each argument not belonging to it.

– Suppose E' is not conflict-free. Let $X, Y \in E'$ s.t. X attacks Y. Hence there are X', Y' in E such that $X' \in X \downarrow AS$ and $Y' \in Y \downarrow AS$ for $AS \subseteq E$ s.t. $cnl(AS) \subseteq \Omega$. Since E is conflict-free, X' does not undercut Y'. Therefore X does not undercut Y. Therefore X rebuts Y. Thus X' also rebuts Y' (at Z). Since E contains all subarguments of its arguments, $Z \in E$ and hence E is inconsistent. From the inconsistency property, E is not conflict-free. Contradiction.
– Let $Z \in AR_{K'} \setminus E'$ s.t. all proper subarguments of Z belong to E'.
We first show that the last rule of Z is defeasible.
Suppose on the contrary, $Z = [Z_1, \dots, Z_n, r]$ where r is a strict rule. From $Z_1, \dots, Z_n \in E'$, it follows that there are $Z'_1, \dots, Z'_n \in E$ and $AS \subseteq E$ s.t. $cnl(AS) \subseteq \Omega$ and $Z'_i \in Z_i \downarrow AS$ for $1 \le i \le n$. Let $Z' = [Z'_1, \dots, Z'_n, r]$. It is clear that $Z' \in Z \downarrow AS$. Since atts is regular, it follows from Lemma 2 and $Z'_1, \dots, Z'_n \in E$ that $Z' \in E$. Therefore $Z \in E'$. Contradiction. Therefore the last rule of Z is defeasible.
From $Z \in AR_{K'}$, it follows that $\exists Z' \in AR_K, AS \subseteq E$ s.t. $cnl(AS) \subseteq \Omega$ and $Z' \in Z \downarrow AS$. Since $Z \notin E'$, it follows that $Z' \notin E$. Therefore there is $A \in E$ s.t. A attacks Z' wrt $atts(K)$. It is obvious A does not attack AS wrt $atts(K)$. Hence from the context-independence property, A attacks Z' wrt $atts(K')$ and A does not attack AS wrt $atts(K')$. Since $atts(K')$ satisfies the link-orientation property, it is clear that A attacks Z wrt $atts(K')$.

The theorem follows from the fact that $cnl(E) = cnl(E')$. □

Appendix F. Extended knowledge bases

Lemma 22. *Let $\mathcal{R}$ be an extended rule-based system and P be a regular property. Further let $\mathcal{S}$ be a non-empty set of attack relation assignments wrt $\mathcal{R}$ satisfying P. Then $\bigcup \mathcal{S}$ also satisfies P.*

Proof. Let $atts_0 = \bigcup \mathcal{S}$.

– Let P be the strong subargument property. We show that $atts_0$ also satisfies the strong subargument property. Let $K \in \mathcal{E}_{\mathcal{R}}$ and $A, B \in AR_K$.
  - It is clear that A attacks B (wrt $atts_0(K)$) iff A attacks B (wrt $atts(K)$) for some $atts \in \mathcal{S}$ iff A attacks a basic defeasible subargument B_0 of B (wrt $atts(K)$) or A attacks an assumption-subargument $[\alpha]$ of B (wrt $atts(K)$) iff A attacks a basic defeasible subargument B_0 of B (wrt $atts_0(K)$) or A attacks an assumption-subargument $[\alpha]$ of B (wrt $atts_0(K)$).
  - It is obvious that if A undercuts or undermines B then A attacks B wrt $atts(K)$ for any $atts \in \mathcal{S}$ and hence A attacks B wrt $atts_0(K)$.
  - It is obvious that if A directly attacks B (wrt $atts_0(K)$) then A directly attacks B (wrt some $atts(K)$, $atts \in \mathcal{S}$) and hence A undercuts or undermines or rebuts B (at B).
– The proofs for other regular properties are identical to the ones in the proof of Lemma 4. □

[30] In other words, E' consists of arguments obtained from arguments $X \in E$ by replacing some subarguments of X by their conclusions provided that the conclusions belong to Ω.

Theorem 13.

1. *For any extended rule-based system $\mathcal{R}$, the normal attack relation assignment $atts_{enr}$ is weakly regular.*
2. *Suppose the extended rule-based system $\mathcal{R}$ satisfies the self-contradiction property. Then the normal attack relation assignment $atts_{enr}$ is regular and the canonical assignment $Ate_{\mathcal{R}}$ exists and $atts_{enr} \subseteq Ate_{\mathcal{R}}$.*

Proof.

1. We show that $atts_{enr}$ is weakly regular. Let $K \in \mathcal{E}_{\mathcal{R}}$.
   (a) It is straightforward to see that $atts_{enr}$ satisfies the property of context-independence. It is also obvious that $atts_{enr}(K)$ satisfies the properties of strong subargument structure.
   (b) We show that $atts_{enr}(K)$ satisfies the property of effective rebut. Let $A_0, A_1 \in AR_K$ such that
   – each A_i, $i = 0, 1$, contains exactly one defeasible rule d_i (i.e. $dr(A_i) = \{d_i\}$), and
   – A_0 rebuts A_1, and
   – A_0 does not undermine A_1.
   Since A_0 neither undercuts nor undermines A_1, it is obvious that A_0 normal-rebuts A_1 iff $d_0 \not\prec d_1$. Hence A_0 attacks A_1 wrt att iff $d_0 \not\prec d_1$.
   (c) We show that $atts_{enr}(K)$ satisfies the property of link-orientation.
   Let $A, B, C \in AR_K$ such that C is a weakening of B by $AS \subseteq AR_K$ (i.e. $C \in B \downarrow AS$) and A does not attack AS wrt $atts_{enr}(K)$ and $(A, C) \in atts_{enr}(K)$. Therefore there are two cases:
   – There is a basic defeasible subargument C' of C such that $cnl(A) = ab_{last(C')}$ or $cnl(A) = \neg cnl(C')$ and there is no $d \in ldr(A)$ s.t. $d \prec last(C')$.
   Since A does not attack AS wrt $atts_{enr}(K)$, the defeasible rule $last(C')$ does not occur in any argument belonging to AS. Hence $last(C')$ occurs in B. Therefore $(A, B) \in atts_{enr}(K)$.
   – $\exists \alpha \in \mathcal{A}$ s.t. $cnl(A) = \overline{\alpha}$ and $[\alpha]$ is subargument of C. Since A does not attack AS wrt $atts_{enr}(K)$, $[\alpha]$ is not a subargument of any argument in AS. Therefore $[\alpha]$ is a subargument of B. Hence $(A, B) \in atts_{enr}(K)$.
   (d) We next show that $att_{enr}(K)$ satisfies the property of attack monotonicity.
   – Let $K \in \mathcal{K}$ and C attacks B wrt $atts_{enr}(K)$ and C is a weakening of A. It is not difficult to see that if C undercuts or undermines B then A also undercuts or undermines B respectively. Suppose now that C rebuts B (at B') and there is no $d \in ldr(C)$ s.t. $d \prec last(B')$. From $ldr(A) \subseteq ldr(C)$ and cnl(C) = cnl(A), it follows obviously that A rebuts B (at B') and there is no $d \in ldr(A)$ s.t. $d \prec last(B')$. We have proved that A also attacks B wrt $att_{enr}(K)$.
   – Suppose A attacks B wrt $atts_{enr}(K)$ and D is a weakening of B. There are two cases:
     - There exists a basic defeasible subargument B' of B such that $last(A) = ab_{last(B')}$ or A normal-rebuts B' (at B'). It is easy to see that there is a subargument D' of D such that D' is a weakening of B'. Hence $last(D') = last(B')$ and D' is basic defeasible. It holds obviously that either $last(A) = ab_{last(D')}$ or A normal-rebuts D' (at D'). A thus attacks D wrt $atts_{enr}(K)$.
     - $\exists \alpha \in \mathcal{A}$ s.t. $cnl(A) = \overline{\alpha}$ and $[\alpha]$ is a subargument of B. It is clear that $[\alpha]$ is also a subargument of D. A hence undermines D. A thus attacks D wrt $atts_{enr}(K)$.
2. To show that $atts_{enr}$ is regular, we need to show that $atts_{enr}(K)$ satisfies the inconsistency-resolving property.
   We first introduce a helpful notation. An assumption argument $[\alpha]$ is said to be maximal assumption-subargument of an argument A iff $[\alpha]$ is an assumption-subargument of A and $[\alpha]$ is not a subargument of any basic defeasible argument of A.
   Let $S \subseteq AR_K$ s.t. S is inconsistent. Let $BE_0 = cnl(S) \setminus \mathcal{A}$. Since S is inconsistent, there are two non-defeasible arguments A_0, A_1 of the knowledge base $(\mathcal{R}, BE_0)$ with contradictory conclusions. Let A'_i, $i = 0, 1$, be weakening of A_i by replacing each subargument $[e]$, $e \in BE_0$, of A_i by an argument in S with conclusion e. Let BE_1 be the set of (non-assumption) premises of arguments A'_0, A'_1. It is clear that $BE_1 \subseteq BE_K$.
   Let $MDA_i = \{X \mid \text{X is a maximal basic defeasible subargument of } A'_i\} \cup \{[\alpha] \mid [\alpha] \text{ is a maximal assumption-subargument of } A'_i\}$ for $i = 0, 1$.
   Since A'_0, A'_1 have contradictory conclusion, it follows immediately that the set $cnl(MDA_0 \cup MDA_1) \cup BE_1$ is inconsistent. Let C be a minimal inconsistent subset of $cnl(MDA_0 \cup MDA_1) \cup BE_1$. Because K is consistent and $BE_1 \subseteq BE_K$, $C \setminus BE_1 \neq \emptyset$.
   Let S_0 be a minimal subset of $MDA_0 \cup MDA_1$ such that $cnl(S_0) = C \setminus BE_1$. S_0 therefore is non-empty and finite. Because S_0 is minimal and rules do not contain non-domain atoms in their bodies, $cnl(S_0)$ does not contain any non-domain atom.
   Let $LD = \{last(X) \mid X \in S_0, X \text{ is basic defeasible}\} \cup \{\alpha \mid \alpha \in \mathcal{A}, [\alpha] \in S_0\}$.
   LD is hence finite and non-empty.
   There are two cases:
   – LD contains an assumptions α. From the self-contradiction property, it holds that $C \vdash \overline{\alpha}$. We could construct an argument B such that $cnl(B) = \overline{\alpha}$ and all maximal basic defeasible subarguments of B as well as all maximal assumption-subarguments of B are arguments in S_0. Therefore B is generated by S and B undermines some argument in S_0. Hence B undermines some argument in S (q.e.d).

– LD contains no assumptions. From the transitivity of $\preceq$, it follows that $\prec$ is a strict partial order. Therefore $\prec$ is a strict partial order on LD. Thus there exists a minimal element wrt $\prec$ in LD. Let $A \in S_0$ s.t. last(A) is minimal (wrt $\prec$). Since $cnl(S_0)$ does not contain any non-domain atom, $hd(last(A)) \in \mathcal{L}_{dom} \setminus \mathcal{A}$. From the self-contradiction property, $C \vdash \neg hd(last(A))$. We could then construct an argument B such that B rebuts A (at A) and all maximal basic defeasible subarguments of B as well as all maximal assumption-subarguments of B are arguments in S_0. Therefore $ldr(B) \subseteq LD$. Since last(A) is minimal (wrt $\prec$) in LD, there is no $d \in ldr(B)$ s.t. $d \prec last(A)$. Therefore B normal-rebuts A, i.e. B attacks A wrt $atts_{enr}(K)$. □

Theorem 14. *Let $\mathcal{R}$ be a well-prioritized extended rule-based system satisfying the self-contradiction property. The canonical attack relation assignment $Ate_{\mathcal{R}}$ and the normal attack relation assignment $atts_{enr}$ coincide.*

Proof. Because $atts_{enr} \subseteq Ate_{\mathcal{R}}$, we only need to show that for each $K \in \mathcal{C}_{\mathcal{R}}$, if $(A, B) \notin atts_{enr}(K)$ then $(A, B) \notin Ate_{\mathcal{R}}(K)$.

Let $K \in \mathcal{C}_{\mathcal{R}}$, $(A, B) \notin atts_{enr}(K)$.

Suppose that $(A, B) \in Ate_{\mathcal{R}}(K)$.

From $(A, B) \notin atts_{enr}(K)$, it is clear that A neither undercuts nor undermines B. From $(A, B) \in Ate_{\mathcal{R}}(K)$, it follows that A rebuts B. From $(A, B) \notin atts_{enr}(K)$, it holds immediately that A does not normal-rebut B.

Therefore for all basic defeasible subarguments Y of B, if A rebuts Y (at Y) then $\exists d_Y \in ldr(A)$ s.t. $d_Y \prec last(Y)$.

From $(A, B) \in Ate_{\mathcal{R}}(K)$ and the regularity of $Ate_{\mathcal{R}}$, A directly attacks a basic defeasible subargument X of B wrt $Ate_{\mathcal{R}}(K)$. Since A neither undercuts B nor undermines B, it follows that A neither undercuts X nor undermines X. From the property of strong subargument structure, it holds directly that A rebuts X (at X). Therefore $d_X \prec last(X)$.

Let BE_0 be the set of evidences from BE_K on which arguments A,X are based, i.e. $BE_0 = \{e \in BE_K \mid [e]$ is a subargument of A or X$\}$.

Let $A_0 = str(A, d)$ and $X_0 = str(X, last(X))$. Let BE_1 be the set of (non-assumption) evidences on which the arguments A_0, X_0 are based.

Since $d_X \prec last(X)$, it follows from the well-prioritizedness of $\mathcal{R}$ that $\Delta(bd(last(X))) \cup \Delta(\neg hd(last(X)))$ is consistent. Therefore $hd(last(X)) \notin \Delta(bd(last(X)))$. Since $cnl(A) = \neg hd(last(X))$, it follows that A does not rebut any proper subargument of X.

From $bd(last(X)) \subseteq \Delta(bd(last(X)))$, it follows $BE' = BE_0 \cup BE_1 \subseteq \Delta(cnl(A)) \cup \Delta(bd(last(X)) = \Delta(\neg hd(last(X))) \cup \Delta(bd(last(X))$.

From the well-prioritizedness of $\mathcal{R}$, BE' is consistent. Therefore $K' = (\mathcal{R}, BE') \in \mathcal{C}_{\mathcal{R}}$ and the arguments A, X, A_0, X_0 belong to $AR_{K'}$. From the context-independence property, it follows that A attacks X wrt $Ate_{\mathcal{R}}(K')$.

We have proved that A attacks X wrt $Ate_{\mathcal{R}}(K')$, A does not undermine X, A rebuts X (at X), and A does not rebut any proper subargument of X and A, X, A_0, X_0 belong to $AR_{K'}$. From assertion 2 of Lemma 27, it follows that $d_X \not\prec last(X)$. Contradiction.

Therefore the assumption that $(A, B) \in Ate_{\mathcal{R}}(K)$ is false. □

Lemma 27. *Let atts be a regular attack relation assignment for extended rule-based system $\mathcal{R}$. Further let $K \in \mathcal{C}_{\mathcal{R}}$, $A, B \in AR_K$ and $d \in ldr(A)$ such that the following properties hold:*

- *A attacks B (wrt atts(K)).*
- *A does not undermine B.*
- *A rebuts B (at B) and A does not rebut any proper subargument of B.*
- *Both $str(B, last(B))$ and $str(A, d)$ belong to AR_K.*

The following conditions hold:

1. *$str(A, d)$ attacks $str(B, last(B))$ (wrt atts(K)).*
2. *$d \not\prec last(B)$.*

Proof.

1. From the attack monotonicity, $str(A, d)$ attacks B. Since A rebuts B, A does not undercut B. Therefore A neither undercuts not undermines B. Thus $str(A, d)$ neither undercut not undermines B. It is also clear that $str(A, d)$ rebuts B and does not rebut any proper subargument of B. Therefore $str(A, d)$ does not attack any proper subargument of B. Therefore from the link-orientation property, $str(A, d)$ attacks $str(B, last(B))$.
2. Since atts is regular and $str(A, d)$ attacks $str(B, last(B))$ and $str(A, d)$ does not undermine $str(B, last(B))$, the effective rebut property directly implies $d \not\prec last(B)$. □

Theorem 15. *Suppose the extended rule-based system $\mathcal{R}$ satisfies the property of self-contradiction. Then for each $K \in \mathcal{C}_{\mathcal{R}}$, each stable extension wrt $atts_{enr}(K)$ is also a stable extension wrt $Ate_{\mathcal{R}}(K)$ and vice versa.*

Proof. From Theorem 13, the canonical attack relation assignment $Ate_{\mathcal{R}}$ exists. Since $atts_{enr}(K) \subseteq Ate_{\mathcal{R}}(K)$, every stable extension wrt $atts_{enr}(K)$ is a stable extension wrt $Ate_{\mathcal{R}}(K)$ (Lemma 28).

Let *atts* be a regular attack relation assignment of $\mathcal{R}$. Further let $K = (\mathcal{R}, BE) \in \mathcal{C}_{\mathcal{R}}$ and E be a stable extension of $(AR_K, atts(K))$.

To show that every stable extension wrt $Ate_{\mathcal{R}}(K)$ is a stable extension wrt $atts_{enr}(K)$, we show that E is a stable extension wrt $atts_{enr}(K)$.

- From Theorem 13, Lemma 21 and Lemma 28, it is clear that E is conflict-free wrt $atts_{enr}(K)$.
- We show that E attacks (wrt $atts_{enr}(K)$) each argument not belonging to E. Let $B \in AR_K \setminus E$. Without loss of generality, we can assume that all proper subarguments of B belong to E (otherwise just pick a subargument of B satisfying this property). It follows immediately that B is either a basic defeasible argument or an assumption-argument (otherwise B would be generated by its basic defeasible subarguments and its assumption-subarguments belonging to E and hence B also belongs to E).
  Since each proper subargument of B belongs to E, no proper subargument of B is rebutted by any argument in E.
  Since E is stable wrt atts(K), there is $A \in E$ s.t. A attacks B wrt atts(K).
  If A undercuts or undermines B then A attacks B (wrt $atts_{enr}(K)$).
  Suppose that A rebuts B and A does not undermine B. Therefore A rebuts B (at B) and A does not rebut any proper subargument of B.
  If A is strict then it is obvious that $(A, B) \in atts_{enr}(K)$. Hence E attacks B (wrt $atts_{enr}(K)$).
  Suppose A is defeasible. Let S be the set consisting of all (non-assumption) evidences on which all arguments of the form $str(A, d)$, $d \in ldr(A)$ and $str(B, last(B))$ are based. It is clear that $S \subseteq cnl(E)$. Let $BE' = BE_K \cup S$. Since $BE_K \subseteq cnl(E)$, it is clear $BE' \subseteq cnl(E)$. Therefore $K' = (\mathcal{R}, BE') \in \mathcal{C}_{\mathcal{R}}$. Therefore $str(A, d)$, $d \in ldr(A)$ and $str(B, last(B))$ all belong to $AR_{K'}$.
  Since atts is regular and hence satisfies the context-independence property, A attacks B wrt $atts(K')$. We have proved that A rebuts B (at B) and A does not rebut any proper subargument of B and A does not undermine B. Further for each $d \in ldr(A)$, both $str(A, d)$ and $str(B, last(B))$ all belong to $AR_{K'}$.
  From Lemma 27, it follows that there is no $d \in ldr(A)$ s.t. $d \prec last(B)$. Therefore A normal-rebuts B, i.e. A attacks B wrt normal attack relation $atts_{enr}(K)$.

We have proved that E is stable wrt $atts_{enr}(K)$. □

Lemma 28. *Let $atts, atts'$ be regular attack relation assignments for an extended rule-based system $\mathcal{R}$.*

1. *Let $K \in \mathcal{C}_{\mathcal{R}}$ and $S \subseteq AR_K$ such that S contains all arguments generated from its arguments. Then S is conflict-free wrt $atts(K)$ iff S is also conflict-free wrt $atts'(K)$.*
2. *Suppose $atts \subseteq atts'$. Then*
   (a) *each stable extension of $(AR_K, atts(K))$ is a stable extension of $(AR_K, atts'(K))$; and*
   (b) *each stable extension of $(AR_K, atts(K))$ is a stable extension of $(AR_K, Ate_{\mathcal{R}}(K))$.*

Proof.

1. Suppose S is conflict-free wrt $atts(K)$ and S is not conflict-free wrt $atts'(K)$. Since $atts(K)$ and $atts'(K)$ have the same set of undercuts and undermines, some argument in S rebuts another. Since all subarguments of arguments in S belong to S, S is inconsistent. Because $atts(K)$ satisfies the inconsistency-resolving property, some argument in S is attacked (wrt atts(K)) by some generated by S. Since S contains all arguments generated from its arguments, S is not conflict-free wrt atts(K). Contradiction. Therefore S is also conflict-free wrt $atts'(K)$.
2. Identical to the proof of Lemma 17. □

Appendix G. Infimum of regular attack relation assignments

We only need to show that *atts* is regular as the proof of the regularity of $atts'$ is identical.
It is obvious that *atts* satisfies the properties of context-independence, effective rebut.
Let $K \in \mathcal{C}_{\mathcal{R}}$.

1. We show that $atts(K)$ satisfies the property of inconsistency-resolving.
   Let $S \subseteq AR_K$ be inconsistent. Since $\mathcal{R}$ contains no strict rule, there are two arguments $X, Y \in S$ with contradictory conclusions. Since K is consistent and $\mathcal{R}$ contains no strict rule, one of X, Y is basic defeasible. Let Y be basic defeasible. Therefore X rebuts Y (at Y).
   If $X \neq D$ then from Lemma 29 (below), $(X, Y) \in atts(K)$.
   Suppose $X = D$. Since D rebuts Y (at Y), Y rebuts D (at D). It is obvious $Y \neq D$. From Lemma 29, $(Y, D) \in atts(K)$.

2. We show that $atts(K)$ satisfies the property of link-orientation.
   Let $X, Y, Z \in AR_K$ such that Z is a weakening of Y by $AS \subseteq AR_K$ and X attacks Z (wrt $atts(K)$) and X does not attack any argument in AS (wrt $atts(K)$). We show that X attacks Y (wrt $atts(K)$).
   From $(X, Z) \in atts(K)$ and Lemma 29, it follows that $X \neq D$ and X rebuts Z.
   Since X does not attack any argument in AS (wrt $atts(K)$) and $X \neq D$, it follows from Lemma 29 that X does not rebut any argument in AS.
   Therefore X rebuts Y. From $X \neq D$ and X rebuts Y, it follows $(X, Y) \in atts(K)$ (Lemma 29).
3. We show that $atts(K)$ satisfies the property of attack-monotonicity.
   Let $X, X', Y, Y' \in AR_K$ such that X' is a weakening of X and Y' is a weakening of Y.
   - Suppose $(X, Y) \in atts(K)$. We show that $(X, Y') \in atts(K)$.
     From Lemma 29, $X \neq D$ and X rebuts Y. Therefore X rebuts Y'. It follows from Lemma 29 that $(X, Y') \in atts(K)$ holds.
   - Suppose $(X', Y) \in atts(K)$. We show that $(X, Y) \in atts(K)$.
     From Lemma 29, $X' \neq D$ and X' rebuts Y. Therefore X rebuts Y.
     We show that $X \neq D$. Suppose on the contrary $X = D$. Because X' is a weakening of X and $\mathcal{R}$ has no rule with head d, it follows immediately that $X' = X = D$. Contradiction.
     We have proved $X \neq D$. It follows from Lemma 29 that $(X, Y) \in atts(K)$ holds.
4. We show that $atts(K)$ satisfies the property of strong subargument structure.
   Let $X, Y \in AR_K$.
   - We first show that X attacks Y (wrt $atts(K)$) iff X attacks a basic defeasible subargument of Y (wrt $atts(K)$).
     - Suppose X attacks Y (wrt $atts(K)$). Therefore $X \neq D$ and X rebuts Y (at Y'). Therefore $(X, Y') \in atts(K)$ and Y' is a basic defeasible subargument.
     - Suppose X attacks a basic defeasible subargument Y' of Y (wrt $atts(K)$). Therefore $X \neq D$ and X rebuts Y'. Therefore X rebuts Y. From Lemma 29, $(X, Y) \in atts(K)$.
   - We show that if X directly attacks Y (wrt $atts(K)$) then X rebuts Y (at Y).
     From Lemma 29, it follows $X \neq D$ and X rebuts Y. Since X does not attack any proper subargument of Y (wrt $atts(K)$), it follows from Lemma 29 that X does not rebut any proper subargument of Y. Therefore X rebuts Y (at Y).

Lemma 29. *Let $K \in \mathcal{C}_{\mathcal{R}}$ and $X, Y \in AR_K$. The following assertions hold.*

1. *X rebuts Y iff $(X, Y) \in Batts(K)$.*
2. *$(X, Y) \in atts(K)$ iff $X \neq D$ and X rebuts Y.*
3. *Let $X \neq D$. $(X, Y) \in atts(K)$ iff X rebuts Y.*

Proof. Assertion 1 is obvious. Assertion 3 follows immediately from assertion 2. We prove assertion 2.
Let $(X, Y) \in atts(K) = Batts(K) \setminus \{(D, A), (D, B)\}$.
Suppose $X = D$. Therefore D rebuts Y. Hence $Y \in \{A, B\}$. Contradiction. From assertion 1, it is clear that X rebuts Y.
The other direction follows obviously from the definition of atts. □

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