

Inductive Defense For Sceptical Semantics of Extended Argumentation

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Abstract

An abstract argumentation framework may have many extensions. Which extension should be adopted as the semantics depends on the sceptical attitudes of the reasoners. Different degrees of scepticism lead to different semantics ranging from the grounded extension as the most sceptical semantics to preferred extensions as the least sceptical semantics. Extending abstract argumentation to allow attacks to be attacked, subjects attacks to argumentation and hence gives rise to a new dimension of scepticism for characterizing how sceptically attacks are accepted. In this paper we present a semantics based on the notion of inductive (grounded) defense of attacks which is sceptical towards the acceptance of attacks but credulous towards the acceptance of arguments. We show that the semantics preserves fundamental properties of abstract argumentation including the monotonicity of the characteristic function. We further show that any extension of the semantics proposed by Gabbay; Baroni, Cerutti, Giacomin and Guida¹; and Modgil contains a sceptical part being an extension of our semantics, and a credulous part resulted from its credulousness towards the acceptance of attacks. We then introduce a stratified form of extended argumentation which still allows an unbounded number of levels of attacks against attacks while assuring that all proposed semantics coincide. In this paper we also develop a sound and complete dialectical proof procedure for the presented semantics following a model of dispute that alternates between argumentation to accept arguments and to accept attacks.

¹Baroni et al for short.

1 Introduction

Argumentation is a form of reasoning, that could be viewed as a dispute resolution, in which the participants present their arguments to establish, defend or attack certain propositions. Argumentation provides a basis for understanding non-monotonic and defeasible reasoning [7, 6, 12, 13, 14, 21, 22, 23, 27, 28, 29, 38, 40], a promising platform for investigating decision making, dialog, negotiation, legal reasoning, learning, and dispute resolution [15, 16, 17, 19, 24, 20, 30, 31, 35, 37, 39, 41, 42].

Abstract argumentation [21] provides a bridge between argumentation theory and application of argumentation models in various directions. An abstract argumentation framework is defined by a set of arguments together with a binary relation representing the attack relationship between arguments. The semantics of abstract argumentation are based on a notion of “acceptability” of arguments, namely an argument A is acceptable wrt a set S of arguments iff S attacks every arguments attacking A . Several semantics could be defined based on sceptical attitudes of reasoners. Different degrees of scepticism lead to different semantics ranging from grounded extension as the most sceptical semantics, to “ideal” extensions as an “ideally” sceptical semantics, and to preferred extensions as the least sceptical semantics [21].

Abstract argumentation has been extended by a number of authors. Amgoud and Cayrol in [3, 2] introduced a preference relation between arguments, resulting in a preference-based argumentation framework in which an attack (A, B) only succeeds if B is not preferred to A . Bench-Capon in [8, 9] dealt with social values that arguments promote, resulting in value-based argumentation which provides a natural basis for legal case-based reasoning [4, 10, 11]. Cayrol and Lagasquie-Schiex in [18], Amgoud et al. in [3] dealt with a support relation between arguments in bipolar argumentation frameworks. Nielsen and Parsons in [36] dealt with joint attacks of arguments. Recently there are proposals for allowing attacks to be attacked [33, 5, 7, 26]. In the latter line of work, Gabbay [26] and Baroni et al. [5] have given semantics for the most general extension of abstract argumentation (until today), where not only attacks against attacks but also attacks against attacks against attacks and so on are allowed.

Attacks against attacks could be viewed as a special kind of arguments, posing a problem of defense for not only arguments but also associated attacks. The acceptance of attacks under different sceptical attitudes gives rise to a new dimension of scepticism. In this paper we explore this dimension, with a semantics for the general extended argumentation frameworks of Gabbay [26], Baroni et al. [5], Modgil [33]. Our semantics, based on an inductive defense relation, is sceptical and grounded towards the acceptability of at-

tacks in a sense that an attack is “acceptable” wrt a set of arguments S only if it is inductively (or groundedly) defended by S , but could be credulous towards the acceptability of arguments. We show that our semantics preserves the key properties of well established semantics for abstract argumentation, like the Fundamental Lemma and the monotonicity of the characteristic function, addressing also an intriguing problem arising from its non-monotonicity in Modgil’s semantics [33]. We also study relationships between our semantics and other semantics including that of Gabbay [26], Baroni et al. [5], and Modgil [33]. Specifically, generalising the idea in abstract argumentation that every extension contains the unique grounded extension, we show that any extension of other semantics contains a sceptical part being an extension of our semantics, and a credulous part resulted from the credulousness towards the acceptance of attacks.

For illustration consider a framework in Figure 1 consisting of attacks $\alpha_1 = (A, A)$ and $\alpha_{i+1} = (A, \alpha_i)$ for $i \geq 1$.

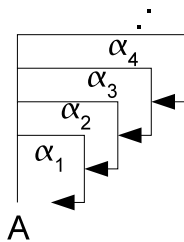


Figure 1. A bizarre framework

It is rather hard to imagine any practical interpretation of this framework. Hence, as a sceptical reasoner, one would not want to draw any conclusion, i.e. does not accept A . An agent arguing for A has to rely on an infinite line of defense $\alpha_2, \alpha_4, \dots$. The semantics of both Gabbay and Baroni et al. has a unique preferred extension $\{A, \alpha_2, \alpha_4, \dots\}$ while our semantics has the empty set as the only extension. The example suggests that extended argumentation in a too literal form would allow counter-intuitive extensions. Thus there arise two problems. The first is to identify from an arbitrary framework a part that is mostly deemed sensible for acceptance, in other words to identify a sceptical generalization to extended abstract argumentation of the semantics for abstract argumentation. Our semantics is developed to address this problem. The second problem is to identify classes of extended argumentation that are appealing for different kinds of well motivated interpretations. To reason with preference among arguments, Modgil in [33] proposed a well-

motivated form of extended argumentation frameworks with only one level of attacks against attacks, generalizing earlier forms proposed by several authors including Amgoud and Cayrol ([1]), Bench-Capon ([9]). Generalizing Modgil’s work, we introduce a class of stratified frameworks where an unbounded number of levels of attacks against attacks against attacks etc, is allowed. The class guarantees that all proposed semantics coincide. Thus, analogous to stratified logic programs, stratified frameworks may be useful for applications including but not limited to reasoning with preferences.

In this paper we also develop a dialectical proof procedure that is sound and complete for our semantics following a model of dispute that alternates between argumentation wrt arguments and argumentation wrt attacks. To our best knowledge the only other proof procedure for extended argumentation in the literature has been proposed by Modgil for his semantics of extended argumentation [32].

At the end we discuss relevant representation issues of extended argumentation, especially the issue of finding an appropriate representation for a problem. We illustrate the issue by discussing intuitive and counter-intuitive interpretations of an example used by Baroni et al. to motivate their semantics.

The structure of the paper is as follows. In section 2, we recall the extended argumentation framework from [26]. In section 3, we introduce our semantics. In section 4, we elaborate on the sceptical nature of our semantics. In section 5, we develop a dialectical proof procedure for computing our semantics. In section 6, we compare our semantics with other proposals. In section 7 we introduce a class of stratified extended argumentation frameworks and show that all proposed semantics coincide for this class. Finally we conclude in section 8.

2 Extended Argumentation Framework

An abstract argumentation framework [21] is a pair $AAF = (AR, Att)$, where AR is a set of arguments, and Att is a binary relation over AR representing the attack relation between the arguments ($Att \subseteq AR \times AR$) with $(A, B) \in Att$ meaning A attacks B . A set S of arguments attacks an argument A if some argument in S attacks A ; S attacks another set S' if S attacks some argument in S' .

A set S of arguments is *conflict-free* iff it does not attack itself. Argument A is *acceptable* with respect to S iff S attacks each argument attacking A . S is *admissible* iff S is conflict-free and each argument in S is acceptable with respect to S . S is a *preferred extension* iff S is maximally (wrt set inclusion)

admissible.

The semantics of argumentation could also be characterized by a fixpoint theory of the characteristic function $F(S) = \{A \in A \mid A \text{ is acceptable wrt } S\}$. It is easy to see that S is admissible iff S is conflict free and $S \subseteq F(S)$. As F is monotonic, it follows that S is a preferred extension iff S is a maximal (wrt set inclusion) fixed point of F . A complete extension is a fixed point of F . The *grounded extension* is the least complete extension.

Which extension is accepted by a reasoner as the semantics of an argumentation framework depends on her degree of scepticism with the grounded extension represents the most sceptical semantics and preferred extensions represent the least sceptical ones.

Among the most general (until today) extended forms of abstract argumentation are the forms proposed in [5, 7, 26, 36] where not only attacks against attacks but also attacks against attacks against attacks and so on are allowed. An attack can come from an argument, or an attack (as in [7]), or a set of them (as in [36]). We recall the following definition from [26].²

Definition 2.1 *An Extended Argumentation Framework (EAF) is a pair (AR, Att) where:*

- AR is a set of arguments;
- Att is a set of attacks s.t. $Att = \bigcup_{i=0}^{\infty} Att^i$, where
 - $Att^0 \subseteq AR \times AR$,
 - $Att^{i+1} \subseteq AR \times Att^i$

If $(A, \alpha) \in Att$, we say that A attacks α .

Note that an AAF is an EAF with $Att = Att^0$. An EAF in [33] has one level of attacks against attacks, i.e. $Att = Att^0 \cup Att^1$.

For illustration we borrow the following example from [33].

Example 2.1 *Consider two persons P and Q arguing about weather forecast:*

P: “Today will be dry in London since the BBC forecast sunshine” (A).

Q: “Today will be wet in London since CNN forecast rain” (B).

P: “But the BBC are more trustworthy than CNN” (C).

²In this paper we consider only attacks coming from arguments.

Q: “However, statistically CNN are more accurate forecasters than the BBC” (C').

Q: “And basing a comparison on statistics is more rigorous and rational than basing a comparison on your instincts about their relative trustworthiness” (E).

A and B claim contradictory conclusions and hence attack each other. C and C' are arguments that express preferences for A over B and B over A respectively. C hence attacks the attack from B against A . Similarly C' attacks the attack from A against B . These preferences are contradictory, so C and C' attack each other. At last E states that C' is preferred to C .³

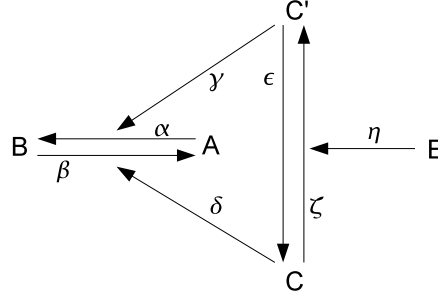


Figure 2. EAF example

The above debate can be represented by the extended argumentation framework in Figure 2 where:

- $AR = \{A, B, C, C', E\}$
- $Att = Att^0 \cup Att^1$, where
 - $Att^0 = \{\alpha, \beta, \epsilon, \zeta\}$ with $\alpha = (A, B)$, $\beta = (B, A)$, $\epsilon = (C', C)$ and $\zeta = (C, C')$
 - $Att^1 = \{\gamma, \delta, \eta\}$ with $\gamma = (C', \alpha)$, $\delta = (C, \beta)$ and $\eta = (E, \zeta)$.

An EAF (AR, Att) is *bounded* if each argument or attack has a finite number of attacks against it, i.e. for each $X \in AR \cup Att$, the set $Attack_X = \{(A, X) \mid (A, X) \in Att\}$ is finite.

From now on, for simplicity **we restrict ourselves to bounded EAFs** and we always refer to an arbitrary but fixed bounded EAF (AR, Att) if not explicitly mentioned otherwise.⁴

³An attack (A, B) , $B \in AR$ is represented by an arrow from A to B . An attack (A, α) , $\alpha \in Att$ is represented by an arrow from A to the arrow representing α .

⁴The results in this paper could be easily generalized to the general case of unbounded EAFs.

3 Inductive Semantics of EAFs

Attacks against attacks pose a problem for defense of not only arguments but also associated attacks. The notion of inductive defense below captures a sceptical attitude of rational agents towards the acceptance of attacks.

Definition 3.1 *Let S be a set of arguments and $\beta \in Att$. We say that*

- S *i-defends*⁵ β *within 0-steps iff there is no argument $C \in AR$ s.t. C attacks β .*
- S *i-defends* β *within $(k+1)$ -steps iff S i-defends β within k -steps or for each argument C if C attacks β then there is a $D \in S$ s.t.*
 - D attacks C and S *i-defends* (D, C) *within k -steps, or*
 - D attacks (C, β) and S *i-defends* $(D, (C, \beta))$ *within k -steps.*
- *We say that S i-defends an attack β iff there is k such that S i-defends β within k -steps.*

Example 3.1 *Consider the EAF in Figure 3 (borrowed from [5])⁶. Let $S = \{A, P\}$.*

- α, ϵ, δ *are i-defended by S within 0-steps.*
- γ *is i-defended by S within 1-step.*

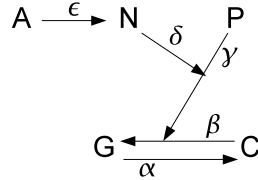


Figure 3. EAF Example

If S *i-defends* α then we often say that α is *i-defended* by S .

Given a set S of arguments, let $\Delta(S)$ denote the set of attacks *i-defended* by S , i.e.

$$\Delta(S) = \{\alpha \in Att \mid \alpha \text{ is i-defended by } S\}$$

The following lemma establishes a monotonic property of the *i-defense* relation.

⁵*i-defends* stands for inductively defends.

⁶We provide an elaborated discussion of intuitive interpretations and counter-intuitive interpretations of this framework in section 8

Lemma 3.1 Given $S, S' \subseteq AR$. If $S \subseteq S'$ then $\Delta(S) \subseteq \Delta(S')$.

Proof: See Appendix A.1 ■

The following definitions generalize the notions of conflict-freeness and acceptability of abstract argumentation.

Definition 3.2 A set S of arguments is ***i*-conflict free** iff there is no $A, B \in S$ s.t. A attacks B and S *i*-defends (A, B) .

Definition 3.3 An argument A is ***i*-acceptable** wrt a set of arguments S iff for each argument B attacking A , there exists $C \in S$ such that:

1. C attacks (B, A) and S *i*-defends $(C, (B, A))$ or
2. C attacks B and S *i*-defends (C, B) .

Example 3.2 (Continue Example 3.1) In Figure 3, G is *i*-acceptable wrt $S = \{A, P\}$ because C attacks G but P attacks (C, G) and S *i*-defends $\gamma = (P, (C, G))$ within 1-steps.

Definition 3.4 An *i*-conflict free set $S \subseteq AR$ is ***i*-admissible** iff every argument in S is *i*-acceptable wrt S .

The following lemma will be useful in relating our notion of *i*-admissibility to the conflict-freeness of Modgil [33].

Lemma 3.2 Let S be an *i*-admissible set of arguments and $A, B \in S$. If A attacks B then there exists $C \in S$ s.t. C attacks (A, B) and S *i*-defends $(C, (A, B))$.

Proof: See Appendix A.2 ■

The characteristic function based on the inductive defense is defined as follows:

$$F_I : 2^{AR} \rightarrow 2^{AR}$$

$$F_I(S) = \{A \in AR \mid A \text{ is } i\text{-acceptable wrt } S\}$$

Note that F_I is defined on every set of arguments including the conflicting ones.

Example 3.3 For the EAF in Figure 3: $F_I(\emptyset) = \{A, P\}$, $F_I^i(\emptyset) = \{A, P, G\}$ for $i \geq 2$.

Our semantics preserves key properties of well established semantics for abstract argumentation.

Lemma 3.3 F_I is monotonic (wrt set inclusion).

Proof: See Appendix A.3. ■

The Fundamental Lemma ([21]) also holds for EAFs.

Lemma 3.4 Let S be an i -admissible set of arguments and A and A' be arguments which are i -acceptable wrt S . Then

1. $S' = S \cup \{A\}$ is i -admissible.
2. A' is i -acceptable wrt S' .

Proof: See Appendix A.4. ■

Now we define extensions wrt inductive defense relation.

Definition 3.5 Let $S \subseteq AR$ be an i -conflict free set of arguments.

- S is an **i -preferred extension** iff it is a maximally i -admissible (wrt set inclusion) set.
- S is an **i -complete extension** iff S is i -admissible and each argument which is i -acceptable wrt S belongs to S .
- A set S is the **i -grounded extension** iff it is the least fixed point of F_I .

Note that due to monotonicity of F_I , the i -grounded extension always exists and is the least (wrt set inclusion) i -complete extension.

Example 3.4 For the EAF in Figure 3, $\{A, P, G\}$ is the unique i -preferred, i -complete and i -grounded extension.

The relations between extensions are given in the following theorem.

Theorem 3.1

Each i -preferred extension is an i -complete extension, but not vice versa.

Proof: See Appendix A.5. ■

4 Inductive Semantics as Sceptical Semantics of EAFs

As for abstract argumentation, different semantics for extended argumentation can be defined to capture different degrees of scepticism. Attacks against attacks introduce an extra dimension of scepticism towards the acceptance of attacks. In this section and section 6, we compare the inductive semantics with other proposals, and show that any extension of other semantics consists of a sceptical part being an extension of the inductive defense semantics, and a credulous part resulted from the credulousness towards the acceptance of attacks.

Introduction to Semantics of Gabbay

Partly inspired by the earlier version of this paper, Gabbay in [26] has introduced a general semantics for extended argumentation. To facilitate the comparison, we first recall it here with some modifications.

A set of arguments/attacks is conflict free in [26] if it does not contain an attack together with both its source and target, i.e.:

Definition 4.1 [26] *A set $S \subseteq AR \cup Att$ is **g-conflict-free** iff there are no A, X such that $A, (A, X), X$ are all in S .*

Example 4.1 *In Figure 3, $\{A, N, P\}$ is g-conflict free, but $\{A, N, \epsilon, P\}$ is not g-conflict-free because of triple $A, \epsilon = (A, N)$ and N .*

Definition 4.2 *An argument/attack $X \in AR \cup Att$ is **g-acceptable** wrt a set $S \subseteq AR \cup Att$ iff for each argument A attacking X (i.e. $(A, X) \in Att$), there exists $B \in S$ such that:⁷*

1. $(B, (A, X)) \in S$ or
2. $(B, A) \in S$.

Since g-acceptability (and as will be shown later, bcgg-acceptability of Baroni et al. as well) does not distinguish between arguments and attacks, the insight that attacks could be given a degree of scepticism different to that of arguments, is hidden.

⁷In [26] X is acceptable wrt S if for any $A \in AR$ s.t. $(A, X) \in Att$ there is a $B \in S$ s.t. $(B, A) \in S$ and $B \in S$. We believe that our definition here is what the author means.

Example 4.2 In Figure 3, attack γ is g -acceptable wrt $\{A, \epsilon\}$ but not g -acceptable wrt $\{A\}$.

Note that even though N is not g -acceptable wrt $\{A, \epsilon\}$, but attack δ is g -acceptable wrt $\{A, \epsilon\}$. In other words, rejection of an argument does not mean rejection of attacks emanating from it. Like our semantics, Gabbay's semantics is therefore different from that of Baroni et al. in [5] (See more in a later discussion).

Extensions wrt g -acceptability is defined as follows.

Definition 4.3 Let $S \subseteq AR \cup Att$ be a g -conflict free set of arguments/attacks.

- S is **g -admissible** iff every argument/attack in S is g -acceptable wrt S .
- S is an **g -preferred extension** iff it is a maximally g -admissible (wrt set inclusion) set.
- S is an **g -complete extension** iff S is g -admissible and each argument/attack which is g -acceptable wrt S belongs to S .
- S is the **g -grounded extension** iff it is the least complete extension wrt set inclusion.

Example 4.3 $\{A, P, \epsilon, \delta, \alpha, \gamma, G\}$ is the only g -preferred, g -complete, g -grounded extension of the EAF in Figure 3.

In [26], Gabbay did not provide technical results on the relationships of the different extensions of his semantics. In the following section we prove a few key results and compare Gabbay's semantics to ours.

Relationships between Inductive Defense and Gabbay's Semantics

We now define the characteristic function based on g -acceptability and show its key properties.

$$F_G : 2^{AR \cup Att} \rightarrow 2^{AR \cup Att}$$

$$F_G(S) = \{X \in AR \cup Att \mid X \text{ is } g\text{-acceptable wrt } S\}$$

Example 4.4 Consider the EAF in Figure 3.

$$F_G(\emptyset) = \{A, P, \epsilon, \delta, \alpha\}.$$

$$F_G^2(\emptyset) = \{A, P, \epsilon, \delta, \alpha, \gamma\}.$$

$$F_G^3(\emptyset) = \{A, P, \epsilon, \delta, \alpha, \gamma, G\}.$$

$$F_G^4(\emptyset) = F_G^3(\emptyset).$$

Lemma 4.1 F_G is monotonic (wrt set inclusion).

Proof: See Appendix B.1. ■

The Fundamental Lemma also holds for general semantics of EAFs.

Lemma 4.2 Let S be a g -admissible set of arguments/attacks and X and X' be arguments/attacks which are g -acceptable wrt S . Then

1. $S' = S \cup \{X\}$ is g -admissible.
2. X' is g -acceptable wrt S' .

Proof: See Appendix B.2 ■

The following lemma shows the relationship between i -admissibility and g -admissibility.

Lemma 4.3 A set $S \subseteq AR$ of arguments is i -admissible iff $S \cup \Delta(S)$ is g -admissible.

Proof: See Appendix B.3. ■

Theorem 4.1 For each $S \subseteq AR$, S is an i -complete extension iff $S \cup \Delta(S)$ is a g -complete extension.

Proof: See Appendix B.4. ■

Note that there may exist $R \subseteq AR \cup Att$ that is a g -complete extension but the restriction of $R \cap AR$ is not an i -complete extension, as illustrated by extension S_3 in the following example.

Example 4.5 Consider the EAF in Figure 4 (borrowed from [33]).

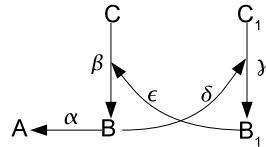


Figure 4. EAF in Example 4.5

The EAF in Figure 4 has the following g -complete extensions:

- $S_1 = \{C, C_1, \alpha, \epsilon, \delta\}$.
- $S_2 = \{C, C_1, B, B_1, \alpha, \epsilon, \delta\}$.
- $S_3 = \{C, C_1, A, \alpha, \beta, \gamma, \epsilon, \delta\}$.

S_1 is the g -grounded extension, S_2 and S_3 are g -preferred extensions.

There are two i -complete extensions:

- $R_1 = \{C, C_1\}$.
- $R_2 = \{C, C_1, B, B_1\}$

R_1 is the i -grounded extension and R_2 is the only i -preferred extension.

It is easy to see that $S_1 = R_1 \cup \Delta(R_1)$ and $S_2 = R_2 \cup \Delta(R_2)$. $S_3 \cap AR = \{C, C_1, A\}$ is not an i -complete extension.

Inductive defense semantics could be viewed as a sceptical approach to the semantics of EAFs. This could be seen by the following theorems.

Theorem 4.2 *Let $S \subseteq AR$ be a set of arguments. S is the i -grounded extension iff $S \cup \Delta(S)$ is the g -grounded extension.*

Proof: See Appendix B.5. ■

The following theorem states that the truly sceptical part of any g -complete extension can be characterized by an i -complete extension.

Theorem 4.3 *Any g -complete extension R contains a greatest (wrt set inclusion) i -complete extension S , i.e. $S \subseteq R$ and for any i -complete extension S' , if $S' \subseteq R$ then $S' \subseteq S$.*

Proof: See Appendix B.6. ■

Example 4.6 (Continuation of Example 4.5) *In Example 4.5, greatest i -complete extensions for g -complete extensions S_1 , S_2 and S_3 are R_1 , R_2 and R_1 respectively.*

The difference $R \setminus S$ is clearly resulted from the credulousness wrt the acceptance of attacks. Its elements are not always sensible, as shown by the example below.

Example 4.7 *For the framework in Figure 1, $R = \{A, \alpha_2, \alpha_4, \dots\}$ is a unique g -preferred extension. The i -complete subset of R is \emptyset .*

Note that in section 6 we show that the relationship between our semantics and that of Baroni et al. and Modgil can be characterized by analogous theorems.

5 A Proof Procedure for i–defense Semantics of EAFs

We present in this section a dialectical proof procedure for i-defense semantics. To our best knowledge the only other proof procedure for extended argumentation in the literature has been proposed by Modgil for his semantics. However there is an extensive research on dialectical proof procedures for abstract argumentation (Vreeswijk and Prakken [41], Cayrol et al. [17], Dung et al. [23, 22], Modgil and Caminada [34], Thang et al. [40], Dung [25], etc.). Our proof procedure is based on two unified frameworks of dialectical proof procedures, respectively of Dung and Thang in [25] for representation and verification of soundness and completeness, and of Thang, Dung and Hung in [40] for development and implementation.

The acceptability of arguments is evaluated by resolving disputes between two fictitious players, a proponent and an opponent. The proponent starts a dispute by putting forward an initial argument, then the proponent and the opponent alternate in attacking each other’s previous arguments and the associated attacks. The proponent wins if the opponent runs out of arguments to make a move. A dispute is represented by a dispute derivation in which tuples $t_i = \langle P_i, O_i, SP_i, SO_i \rangle$ summarizing the history of the dispute up to step i are successively constructed by expanding the previous one. $P_i \subseteq AR \cup Att$ is a set of arguments and attacks presented by the proponent that have not been defended by the proponent and hence are open to attacks by the opponent. $SP_i \subseteq AR$ is the set of all arguments presented by the proponent. Hence the proponent does not need to re-defend arguments in $SP_i \setminus P_i$. $O_i \subseteq Att$ is a set of attacks of the opponent against arguments presented by the proponent in previous steps that are not counter-attacked yet by the proponent. Thus an attack $(B, C) \in O_i$ needs to be counter-attacked by the proponent on either B or (B, C) . SO_i contains attacks by the opponent that have been counter-attacked by the proponent.

In a framework $EAF = (AR, Att)$, for $X \in AR \cup Att$, note that $Attack_X = \{(A, X) \mid (A, X) \in Att\}$.

Definition 5.1 *Given a selection function, a dispute derivation for an argument A is a sequence of tuples:*

$$\langle P_0, O_0, SP_0, SO_0 \rangle \dots \langle P_n, O_n, SP_n, SO_n \rangle$$

where

1. $P_i \subseteq AR \cup Att$; $SP_i \subseteq AR$, and $O_i, SO_i \subseteq Att$.

2. $P_0 = SP_0 = \{A\}$ and $O_0 = SO_0 = P_n = O_n = \emptyset$.

3. At step i , let X be an element selected from either P_i or O_i

(a) If $X \in P_i$ then:

$$P_{i+1} = P_i \setminus \{X\}$$

$$O_{i+1} = O_i \cup \text{Attack}_X$$

$$SP_{i+1} = SP_i$$

$$SO_{i+1} = SO_i$$

(b) If $X = (B, \alpha) \in O_i$, then there exists some attack $(D, Y) \in \text{Att}$ such that

- If $B \notin SP_i$ then $(D, Y) \in (\text{Attack}_B \cup \text{Attack}_{(B, \alpha)}) \setminus (SO_i \cup O_i)$ ⁸

- If $B \in SP_i$ then $(D, Y) \in \text{Attack}_{(B, \alpha)} \setminus (SO_i \cup O_i)$

and:

$$P_{i+1} = P_i \cup \{D, (D, Y)\} \text{ if } D \notin SP_i, \text{ otherwise } P_{i+1} = P_i \cup \{(D, Y)\}$$

$$O_{i+1} = O_i \setminus \{X\}$$

$$SP_{i+1} = SP_i \cup \{D\}$$

$$SO_{i+1} = SO_i \cup \{X\}$$

Example 5.1 (Continue example 4.5) A dispute derivation for B is constructed in Figure 5, where the notation \underline{X} means that X is selected. It shows that B is i -acceptable wrt its constructed i -admissible set $\{B, B_1\}$.

The following theorem states that dispute derivations represent sound and complete proofs for i -admissibility.

Theorem 5.1 (Soundness and Completeness)

1. *Soundness:* If $\langle P_0, O_0, SP_0, SO_0 \rangle \dots \langle P_n, O_n, SP_n, SO_n \rangle$ is a dispute derivation for argument A , then SP_n is i -admissible and contains A .
2. *Completeness:* Let EAF be a finite argumentation framework, and let A be an argument of EAF . If A belongs to an i -admissible set S of arguments, then for any selection function there is a dispute derivation for A , whose component SP_n of the final tuple is a subset of S .

⁸It follows that $Y = B$ or $Y = (B, \alpha)$.

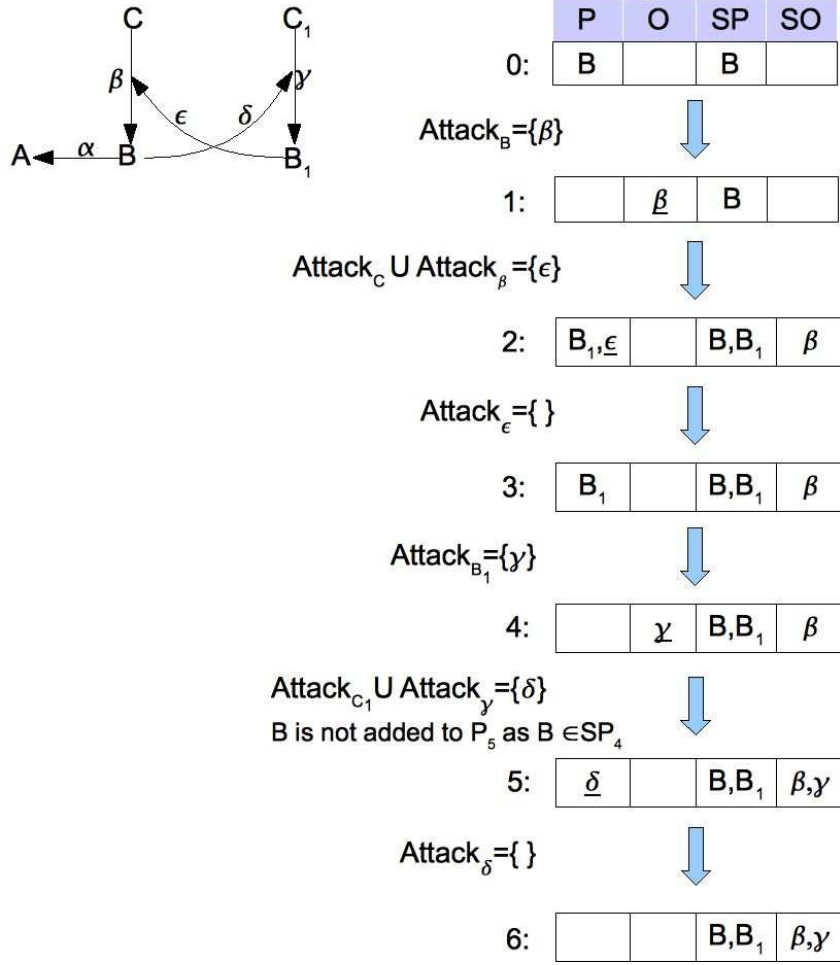


Figure 5. A dispute derivation for B

Proof: See Appendix C.1. ■

Thus a proof procedure for i-defense semantics can be reduced to a procedure searching for dispute derivations, which could be directly implemented by means of, for example, base derivations defined in [40]. As analysed in [25], in general dispute derivations can be equipped with different filtering mechanisms for two distinct purposes: to guarantee the soundness and completeness of the proof procedure or to improve its efficiency. A proof procedure searching for dispute derivations of Definition 5.1 serves our purpose of having a simple yet correct proof procedure, but it could be inefficient and even may not terminate because it lacks filtering mechanisms for efficiency

(for example, the proof does not terminate for argument A in Fig. 5⁹). These mechanisms can mirror those for abstract argumentation developed in [40] and remain a future work.

6 Relationships to other Semantics

Apart from the approach of Gabbay, there are the proposals of Baroni et al. [5] and Modgil [33]. In this section we show how they agree and differ. We believe it may not be the right question to ask which proposal is the best. It would be more appropriate to identify situations where a proposed semantics is useful/appropriate. We will further the argument put forwards in section 4, that our inductive semantics is an appropriate representation of a sceptical reasoner of an EAF, generalizing naturally the idea of the grounded acceptance in abstract argumentation to the acceptance of attacks in EAFs.

Relationships to Semantics of Baroni, Cerutti, Giacomin and Guida.

In [5] Baroni et al. independently introduced an elegant semantics which differs subtly from that of Gabbay. In [5]:

- If $(A, \alpha) \in Att$ then (A, α) *directly defeats* α .
- If $(A, B), (B, \beta) \in Att$ then (A, B) *indirectly defeats* (B, β) .
- $\alpha \in Att$ defeats $\beta \in AR \cup Att$ if α directly or indirectly defeats β .
- A set $S \subseteq AR \cup Att$ is bccg-conflict-free iff there are no $\alpha, \beta \in S$ s.t. α defeats β .
- $X \in AR \cup Att$ is bccg-acceptable wrt $S \subseteq AR \cup Att$ iff for each $\alpha \in Att$ if α defeats X then there is $\beta \in S$ s.t. β defeats α .
- A set $S \subseteq AR \cup Att$ is bccg-admissible iff it is bccg-conflict-free and each element of S is bccg-acceptable wrt S . A bccg-preferred extension is a maximal (wrt set inclusion) bccg-admissible set.

Example 6.1 Consider the framework in Figure 6

⁹But it is not difficult to prove that for stratified EAF defined in section 7 the procedure terminates

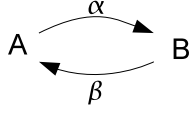


Figure 6. A simple framework

- α (resp. β) defeats B (resp. A) directly but defeats β (resp. α) indirectly.
- $\{\alpha\}, \{\beta\}$ are *bcgg-admissible* and $\{\alpha, A\}, \{\beta, B\}$ are *bcgg-preferred extensions*.
- $\{\alpha, \beta\}$ is not *bcgg-conflict free*, but $\{\alpha, \beta\}$ is *g-conflict free*.

The semantics of Baroni et al. is based on translating an $EAF = (AR, Att)$ into an $AAF = (AR \cup Att, defeat)$. The use of *defeat* leads intuitively to a semantics that an attack is acceptable to a reasoner only if both the attack and its source argument are defensible. This is different to our and Gabbay’s semantics. Consider again the above simple framework, in our and Gabbay’s semantics a reasoner can accept both α and β but in the semantics of Baroni et al. it can not do so.

In [5], Baroni et al. do not explicitly define complete extensions for their semantics. For comparison we introduce them here:

A *bcgg-admissible* set $S \subseteq AR \cup Att$ is a *bcgg-complete extension* if each argument/attack which is *bcgg-acceptable* wrt S belongs to S .

The following lemmas shows how the semantics of Baroni et al. differs from that of Gabbay about the notion of acceptability.

Lemma 6.1 *Given $R \subseteq AR \cup Att$ and $(A, \alpha) \in Att$ and $B \in AR$.*

1. *If both A and (A, α) are *g-acceptable* wrt R , then (A, α) is *bcgg-acceptable* wrt R .*
2. *If B is *g-acceptable* wrt R , then B is *bcgg-acceptable* wrt R .*

Proof: See Appendix D.1. ■

Given $\alpha = (A, \beta) \in Att$, we say α is an attack on β and write $source(\alpha) = A$.

Lemma 6.2 *Let R be a bcgg-complete extension and $\alpha \in Att$. The following properties hold*

1. *If $\alpha \in R$ then $source(\alpha) \in R$.*
2. *$F_G(R) \setminus R \subseteq Att$*
3. *If $\alpha \in F_G(R) \setminus R$ then $source(\alpha) \notin R$.*

Proof: See Appendix D.2. ■

Given a set $R \subseteq AR \cup Att$ of arguments and attacks, let $rd(R)$ denote the subset of R that excludes attacks coming from arguments outside R , i.e.

$$rd(R) = R \setminus \{\alpha \in Att \mid source(\alpha) \notin R\}$$

The following theorem shows the relationship between a g-complete extension and a bcgg-complete extension.

Theorem 6.1 *Given a set $R \subseteq AR \cup Att$ of arguments and attacks.*

1. *If R is g-complete, then $rd(R)$ is bcgg-complete.*
2. *If R is bcgg-complete, then $F_G(R)$ is g-complete.*

Proof: See Appendix D.3. ■

Given a set $S \subseteq AR$ of arguments, let $\Pi(S)$ denote the set of attacks coming from arguments in S and i-defended by S , i.e.

$$\Pi(S) = \{\alpha \in \Delta(S) \mid source(\alpha) \in S\}.$$

The following theorems show the relationship between an i-complete extension and a bcgg-complete extension.

Theorem 6.2 *S is an i-complete extension iff $S \cup \Pi(S)$ is a bcgg-complete extension.*

Proof: See Appendix D.4. ■

Analogous to theorem 4.3, the following one states that the truly sceptical part of any bcgg-complete extension can be characterized by an i-complete extension.

Theorem 6.3 *Any bcgg-complete extension R contains a greatest (wrt set inclusion) i -complete extension S , i.e. $S \subseteq R$ and for any i -complete extension S' , if $S' \subseteq R$ then $S' \subseteq S$.*

Proof: See Appendix D.5. ■

Relationships to Semantics of Modgil

Modgil's extended argumentation framework ([33]) accommodates reasoning with preference among arguments generalizing earlier extensions of the abstract argumentation framework by Amgoud and Cayrol ([1]), Bench-Capon ([9]).

Definition 6.1 *A Modgil's Extended Argumentation Framework (MEAF) is an EAF (AR, Att) with the following constraints:*

1. $Att = Att^0 \cup Att^1$ where $Att^0 \subseteq AR \times AR$ and $Att^1 \subseteq AR \times Att^0$;
2. If both $(Z, (X, Y))$ and $(Z', (Y, X))$ are in Att then both (Z, Z') and (Z', Z) are in Att .

Though the Modgil's extended argumentation could be viewed as a special case of general extended frameworks (including BGW framework [7]¹⁰), its semantics is based on the underlining intuition that attacks against attacks represent preferences between conflicting arguments. Hence the condition 2 in Definition 6.1 is introduced¹¹. This constraint plays a fundamental role in the definition conflict-freeness and hence in Modgil's semantics recalled below. This insight suggests that different intuitions and applications could lead to different classes and different semantics for the general extended argumentation of [7]¹². This line of work [1, 9, 33], however, is important for practical applications as too liberally defined argumentation networks may not have any intuitive interpretation.

In the rest of this chapter whenever we refer to an EAF, we mean an Modgil's extended argumentation framework.

Modgil's semantics ([33]) is also based on a notion of acceptability of arguments wrt a set of arguments. Intuitively, an argument A is acceptable

¹⁰A BGW framework is even more general than the general extended framework studied in this paper as BGW framework allows attacks to come from not only arguments but also attacks.

¹¹An attack $(Z, (X, Y))$ represents the preference Z for Y over X . Contradictory preferences Z and Z' must attack each other.

¹²This view is also shared by Gabbay in [26]

wrt a set of arguments S if S attacks each argument attacking A and there exist reinstatement sets for those attacks from S against attackers of A to ensure that they are reinstated if attacked. We recall the important notions of defeat and reinstatement in [33] below.

Definition 6.2 Let (AR, Att) be a MEAF and $S \subseteq AR$.

- A **defeats** $_S B$ denoted by $A \rightarrow^S B$ iff A attacks B and there is no $C \in S$ s.t. C attacks (A, B) .
- S is **m -conflict free** iff for all $A, B \in S$: if A attacks B then B does not attack A and there is $C \in S$ s.t. C attacks (A, B) .

Definition 6.3 Let (AR, Att) be a MEAF and $S \subseteq AR$.

- $R_S = \{X_1 \rightarrow^S Y_1, \dots, X_n \rightarrow^S Y_n\}$ is a **reinstatement set** for $C \rightarrow^S B$ iff
 1. $C \rightarrow^S B \in R_S$.
 2. For $i = 1 \dots n$: $X_i \in S$.
 3. $\forall X \rightarrow^S Y \in R_S, \forall Y'$ s.t. $(Y', (X, Y)) \in Att$, there is some $X' \rightarrow^S Y' \in R_S$.
- $A \in AR$ is **m -acceptable** wrt S iff for each B s.t. $B \rightarrow^S A$ there is a $C \in S$ s.t. $C \rightarrow^S B$ and there is a reinstatement set for $C \rightarrow^S B$.

Example 6.2 In Figure 4, A is m -acceptable wrt $S = \{C, C_1\}$. The reinstatement set for $C \rightarrow^S B$ is $\{C \rightarrow^S B, C_1 \rightarrow^S B_1\}$.

The semantics of MEAFs are defined by extensions based on Modgil's acceptability as follows.

Definition 6.4 Let S be a m -conflict-free set of arguments.

- S is m -admissible iff each argument in S is m -acceptable wrt S .
- S is a m -preferred extension iff S is a maximal (wrt set inclusion) m -admissible set.
- S is a m -complete extension iff each argument which is m -acceptable wrt S belongs to S .

Example 6.3 In Figure 4, $\{C, C_1, A\}$, $\{C, C_1, B, B_1\}$ are m -preferred extensions.

A is acceptable wrt $\{C, C_1\}$ because of the reinstatement set $\{C \rightarrow^S B, C_1 \rightarrow^S B_1\}$. The meaning of reinstatement in this case is that B and B_1 are defeated by C, C_1 , hence A is acceptable. In other words, the attacks $(C, B), (C_1, B_1)$ are necessary for the acceptability of A wrt $\{C, C_1\}$ because employing them in a reinstatement allows $\{C, C_1\}$ to eliminate the “threats” from B, B_1 to the acceptability of A .

Definition 6.5 Let $EAF = (AR, Att)$ be a MEAF. The Modgil’s *characteristic function* of EAF denoted by F_M is defined as follows:

- $F_M : ARC \rightarrow 2^{AR}$ where ARC is the set of all m -conflict free subsets of AR .
- $F_M(S) = \{A \mid A \text{ is } m\text{-acceptable wrt } S\}$.

In general F_M is not monotonic and the existence of a least fixed point of F_M is not guaranteed.

Example 6.4 In Figure 4 A is m -acceptable wrt $S = \{C, C_1\}$ but not m -acceptable wrt $S' = S \cup \{B, B_1\}$. It means that $S \subseteq S'$ but $F_M(S) \not\subseteq F_M(S')$.

Why F_M is not monotonic? Coming back to the MEAF in Figure 4. A is acceptable wrt $\{C, C_1\}$ but by adding B, B_1 to $\{C, C_1\}$ the attacks (C, B) and (C_1, B_1) are disregarded. Hence they can not be employed to defend A wrt $\{C, C_1, B, B_1\}$ anymore and it leads to the unacceptability of A wrt $\{C, C_1, B, B_1\}$.

Modgil [33] defined the grounded extension of MEAF as the union $\bigcup_{i=0}^{\infty} F_M^i(\emptyset)$. Due to the non-monotonicity of F_M , this grounded extension is not a least complete extension, hence does not represent the most sceptical semantics. It hence does not generalize the idea of the grounded extension as the most sceptical semantics of abstract argumentation. The following example illustrates this point.

Example 6.5 For the framework in Figure 4. From $F_M(\emptyset) = \{C, C_1\}$, $F_M^i(\emptyset) = \{C, C_1, A\}$ for $i \geq 2$, the m -grounded extension is $\{C, C_1, A\}$.

However, since A is not in the other m -preferred extension $\{C, C_1, B, B_1\}$, this m -grounded extension is not a subset of the preferred extension $\{C, C_1, B, B_1\}$. In other words, the “ m -grounded extension” of Modgil does not fully generalize the notion of the grounded acceptance in abstract argumentation as the semantics of the most sceptical reasoners.

There are two different views to the grounded extension:

- as a least complete extension, i.e. it represents the most sceptical semantics of entirely possible semantics acceptable by reasoners.
- as a result of the iterative computation process of the characteristic function from the empty set to yield a fixed point.

In that sense, our grounded semantics captures the most sceptical part of Modgil’s grounded semantics.¹³

We believe that the monotonicity of the characteristic function is a fundamental property of abstract argumentation, establishing a semantics for sceptical reasoners. Further the monotonicity makes it possible, or at least much easier to develop efficient dialectical proof procedures for sceptical reasoners, which rely on the possibility to incrementally constructing proofs as disputes, as illustrated in section 5. The non-monotonicity also runs against the intuition that the more arguments one has, the more arguments one is capable to defend.

The following lemma shows the relationship between *i*-acceptability and Modgil’s *m*-acceptability.

Lemma 6.3 *If an argument A is i -acceptable wrt a m -conflict free set S of arguments then A is m -acceptable wrt S .*

Proof: See Appendix D.6. ■

Lemma 6.3 shows that *i*-admissibility implies *m*-admissibility wrt *m*-conflict free sets. But the reverse is not true as the following example shows. To address this issue we introduce the notion of *mi*-admissibility below.

Example 6.6 *Consider the MEAF in Figure 4.*

A is m -acceptable with $S = \{C, C_1\}$ (Example 6.2). Hence $\{C, C_1, A\}$ is m -admissible.

A is not i -acceptable wrt $\{C, C_1\}$ because (C, B) is not i -defended by $\{C, C_1\}$. Hence $\{C, C_1, A\}$ is not i -admissible.

Example 6.7 *Consider the MEAF in Figure 7.*

The following table shows its extensions.

¹³In [33], Modgil has shown that the characteristic function of hierarchical and preference symmetric frameworks is monotonic, where in the former case, the hierarchical restriction essentially ensures that any defense of an attack is inductive.

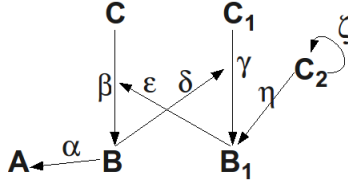


Figure 7. MEAF in Example 6.7

	<i>i</i> -	<i>m</i> -
Preferred	$\{C, C_1\}$	$\{C, C_1, A\}$
Complete	$\{C, C_1\}$	$\{C, C_1, A\}$

Example 6.7 shows that in general:

- An *i*-preferred extension is not necessary a *m*-preferred extension and vice versa.
- An *i*-complete extension is not necessary a *m*-complete extension and vice versa.

Modgil's constraint in his notion of *m*-conflict freeness that there are no two arguments of S attacking each others, is relevant in many practical applications. In the following we incorporate this constraint into our *i*-defense semantics.

Definition 6.6 An *i*-admissible set $S \subseteq AR$ is ***mi*-admissible** iff there are no arguments $A, B \in S$ s.t. A and B attack each other, i.e. $(A, B), (B, A) \in Att$.

Lemma 6.4 Given $S \subseteq AR$. If S is *mi*-admissible, then S is *m*-admissible.

Proof: See Appendix D.7. ■

The following theorem states that *mi*-admissibility could be viewed as a sceptical part in the credulous semantics of Modgil.

Theorem 6.4 Any *m*-preferred extension R contains a greatest (wrt set inclusion) *mi*-admissible set S , i.e. $S \subseteq R$ and for any *mi*-admissible set S' , if $S' \subseteq R$ then $S' \subseteq S$.

Proof: See Appendix D.8. ■

7 Stratified EAFs

Generalizing hierarchical MEAFs [33], we introduce a class of stratified frameworks where an unbounded number of levels of attacks against attacks against attacks, etc, is allowed. We then argue that this class reduces the risk of inferring insensibly in general extended argumentation as all proposed semantics coincide.

For $\alpha = (A, \beta) \in Att$, we define $target(\alpha)$ as follows:

1. If $\beta \in AR$ then $target(\alpha) = \beta$.
2. If $\beta \in Att$ then $target(\alpha) = target(\beta)$.

An $EAF = (AR, Att)$ is *strongly bounded* if for each argument $A \in AR$, the set $\{\alpha \in Att \mid target(\alpha) = A\}$ is finite¹⁴.

Definition 7.1 *An Extended Argumentation Framework $EAF = (AR, Att)$ is stratified iff it is strongly bounded and there exists a partition*

$$AR = \bigcup_{i=0}^{\infty} AR_i$$

such that:

1. AR_i are pairwise disjoint sets of arguments, where $rank(A) = i$ states that $A \in AR_i$,
2. for each $\alpha \in Att$:
 - (a) If $\alpha = (A, B)$ and $A, B \in AR$, then: $rank(A) \leq rank(B)$.
 - (b) If $\alpha = (A, \beta)$ and $\beta \in Att$ then: $rank(A) < rank(target(\beta))$.
3. For any i , if $AR_i = \emptyset$ then for any $j \geq i$, $AR_j = \emptyset$.

The intuition of stratification is that the acceptance of an argument $A \in AR_i$ does not depend on any level AR_j with $j > i$. This property holds when arguments can be ranked such that attacks against arguments come from arguments with smaller or equal ranks, but attacks against attacks come from arguments with strictly smaller ranks.

Example 7.1 *For frameworks we have analysed:*

¹⁴It is easy to see that a strongly bounded EAF is bounded but not vice versa

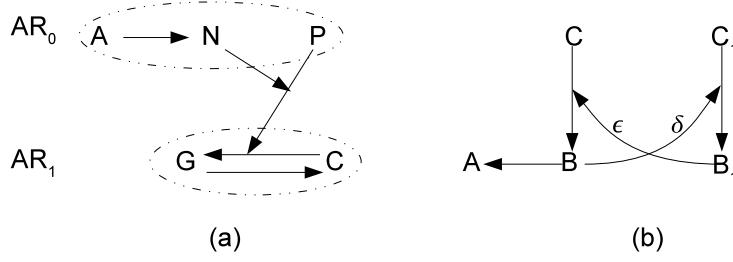


Figure 8. EAF Partitions

- *Figure 8a: The framework is stratified.*
- *Figure 8b: The framework is not stratified. Assume the contrary. Since $source(\delta) = B$ and $target(\delta) = B_1$, $rank(B) < rank(B_1)$. Consider ϵ analogously, $rank(B) > rank(B_1)$. Contradiction !*

Note that a hierarchical framework of Modgil is a stratified framework satisfying $Att = Att^0 \cup Att^1$ and $rank(source(\alpha)) = rank(target(\alpha))$ if $\alpha \in Att^0$ in condition 2a.

Analogous to the class of stratified logic programs, the class of stratified frameworks is useful for applications including but not limited to reasoning with preferences, the motivation of hierarchical frameworks.

Lemma 7.1 *Let $EAF = (AR, Att)$ be a stratified framework and $S \subseteq AR$, $R \subseteq Att$. If $S \cup R$ is g -complete (resp. $bcgg$ -complete), then $R = \Delta(S)$ (resp. $R = \Pi(S)$).*

Proof: See Appendix E.1. ■

Theorem 7.1 *Let $EAF = (AR, Att)$ be a stratified framework and $S \subseteq AR$, $R \subseteq Att$. $S \cup R$ is g -complete (resp. $bcgg$ -complete) iff S is i -complete.*

Proof: See Appendix E.2. ■

Theorem 7.2 *Let $EAF = (AR, Att)$ be a stratified MEAF and $S \subseteq AR$. S is m -complete iff S is i -complete.*

Proof: See Appendix E.3. ■

8 Discussions and Conclusions

This paper focuses on a research line of extending abstract argumentation. Amgoud and Cayrol in [3, 2] introduced a preference relation between arguments, resulting in a preference-based argumentation framework in which an attack (A, B) only succeeds if B is not preferred to A . Bench-Capon in [8, 9] dealt with social values that arguments promote, resulting in value-based argumentation which provides a natural basis for legal case-based reasoning [4, 10, 11]. Cayrol and Lagasquie-Schiex in [18], Amgoud et al. in [3] dealt with a support relation between arguments in bipolar argumentation frameworks. Nielsen and Parsons in [36] dealt with joint attacks of arguments. Recently there are proposals for allowing attacks to be attacked [33, 5, 7, 26]. In the latter line of work, Gabbay [26] and Baroni et al. [5] have given semantics for the most general extension of abstract argumentation (until today), where not only attacks against attacks but also attacks against attacks against attacks and so on are allowed.

While it seems reasonable to expect that attacks can be attacked and so should be subjected to argumentation, it is not always straightforward to find intuitive interpretations for such networks. For illustration we recall an example by Baroni et al. [5] illustrated in Figure 3, where Bob has to decide whether to spend his Christmas holidays in Gstaad, a ski resort, or Cuba, with below arguments:

”There is a last minute offer for Gstaad, so Bob should go to Gstaad” (G).

”There is a last minute offer for Cuba, so Bob should go to Cuba” (C).

”Bob likes skiing, so when it is possible, he prefers to go to a ski resort” (P).

”The weather report informs that in Gstaad there were no snowfalls since one month, so it is not possible to ski in Gstaad” (N).

”Thanks to a good amount of artificial snow in Gstaad, it is anyway possible to ski there” (A).

Baroni et al. built a framework for Bob as follows. As two Bob’s choices are incompatible, G and C attack each other. P represents a preference for skiing and hence attacks the attack from C to G. N does not affect the existence of last minute offers or Bob’s general preferences for ski. Rather, N affects the ability of preference P to affect the choice between Gstaad and Cuba and hence attacks the attack originated from P. However A attacks N, thus reinstating the attack originating from P. The resulted framework suggests that Bob should go to Gstaad.

Does this framework succinctly represent Bob’s dilemma? If the answer is ”Yes”, let us twist it slightly.

Suppose there is no artificial snow. Hence A should be dropped from the

framework. So the attack coming from P should be rejected. Thus for Bob, Cuba and Gstaad are equally preferred. Consider that Gstaad is a ski resort without snow, this conclusion is rather odd. So let us elaborate a bit deeper.

A last minute offer for Gstaad (G) or Cuba (C) is not in itself an argument for Bob to go, but only its abbreviation. A last minute offer to a dangerous place like Afghanistan, may not make sense for Bob. In the context that Bob is deciding about his Christmas holidays, a more complete form of G could be:

- Premise 1: Bob wants to go to places where he can ski.
- Premise 2: Bob can ski in Gstaad since it *normally* has a good amount of snow.
- Premise 3: Bob could accept a last minute offer to Gstaad because he can ski there.
- Conclusion: Bob should go to Gstaad.

Hence instead of attacking the attack originated from P, N attacks G on its second premise, suggesting that Bob should go to Cuba, not Gstaad. This situation is better represented by the framework in Figure 9.

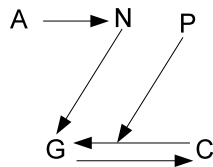


Figure 9. An EAF for Bob's Dilemma Example

That one can have completely different frameworks for the “same problem” exhibit the challenge of representing problems by extended argumentation frameworks. In Bob's dilemma, for example, the framework given by Baroni et al. is appropriate in situations where Bob goes to Gstaad for several amusements, for example skiing and social night life and the value of social life is as good as the value of swimming in Cuba. As also pointed out by Gabbay [26], in extended argumentation it is not easy to say which of rival representations is correct.

Bob's dilemma, and several examples in this paper, reveals that it is a challenge to interpret an extended argumentation framework, especially when it is in a too liberal form. Thus there arises several problems. The first is to identify from an arbitrary framework a part that is deemed sensible

for acceptance. As in abstract argumentation, several semantics could be defined based on sceptical attitudes of reasoners, ranging from grounded extension as the most sceptical semantics, to “ideal” extensions as an “ideally” sceptical semantics, to preferred extensions as the least sceptical semantics. In extended argumentation where both arguments and attacks can be attacked, there are two dimensions of scepticism: one towards the acceptance of arguments, and one towards the acceptance of attacks. In this paper we explore this space with an inductive defense semantics that is sceptical, grounded towards the acceptance of attacks but could be credulous towards the acceptance of arguments. We show that our semantics preserves the key properties of semantics for abstract argumentation (which have only one scepticism dimension), like the Fundamental Lemma and the monotonicity of the characteristic function. These properties play important roles in, for example, developing proof procedures. Demonstratively, we also develop a sound and complete dialectical proof procedure following a model of dispute that alternates between argumentation wrt arguments and argumentation wrt attacks. To our best knowledge the only other proof procedure for extended argumentation in the literature has been proposed by Modgil for his semantics of extended argumentation.

Several semantical systems for extended argumentation have been proposed. Though clearly related, their formal and precise relationship remains much unexplored. We address this problem by providing an unified approach for comparing them based on dimensions of scepticism. We show that any extension of other semantics including that of Gabbay [26], Baroni et al. [5] and Modgil [33] contains a sceptical part being an extension of our semantics, and a credulous part resulted from the credulousness towards acceptance of attacks. As there are two dimensions to scepticism in extended argumentation frameworks, it is also possible to construct a semantics credulous towards the acceptance of attacks while sceptical towards the acceptance of arguments. We believe that this approach could be used to define a sceptical semantics for other extended forms of argumentation like the bipolar argumentation framework of Cayrol et al. [3, 18]. However, for general extended forms like those of Gabbay [26], Baroni et al. [5], it seems prudent for us to start studying with a sceptical approach to acceptance of attacks since we do not know of any example with an intuitive and practical interpretation for credulous acceptance of attacks.

The second problem arising from extending abstract argumentation is to identify classes of extended argumentation frameworks that are appealing to different kinds of well motivated interpretations. In Modgil’s extended argumentation framework [33] which generalizes the work of several authors including Amgoud and Cayrol ([1]), Bench-Capon ([9]), attacks against at-

tacks represent preferences between conflicting arguments. This intuition leads to a constraint that two arguments attacking each other should not be accepted together. The constraint could be imposed directly on the notion of conflict-freeness as in [33] or equivalently on the notion of admissibility as our mi-admissibility¹⁵. We believe that a study of appropriate constraints is important as extended argumentation in a too general form could lead to awkward frameworks with no intuitive meaning. Generalizing Modgil’s work, we introduce a class of stratified frameworks where an unbounded number of levels of attacks against attacks against attacks etc, is allowed, guaranteeing that all proposed semantics coincide. Stratified frameworks may be useful for applications including but not limited to reasoning with preferences.

It is a challenging problem to classify extended argumentation frameworks that are sensible for both sceptical and credulous reasoning. Many ideas for semantics of extended argumentation have been studied in [5, 7, 26, 33] and in this paper. We believe that sensible combination of these ideas could provide sensible semantics for diverse application.

Our contributions can be extended in several directions. The inductive defense semantics can be extended easily for unbounded EAFs (frameworks that against an element there is possibly an infinite number of attacks). By the same proof given in this paper, our proof procedure can be shown to be sound and complete for a general class of finitary frameworks, which require that for an argument/attack, the set of arguments/attacks reachable to it, is finite. Conforming to the framework in [25], our proof procedure is equipped with filtering mechanisms essential for its soundness and completeness. It can also be equipped with other filtering mechanisms for efficiency, for example in its step 3.a, $O_{i+1} = O_i \cup \text{Attack}_X$ could be replaced by $O_{i+1} = O_i \cup (\text{Attack}_X \setminus (SO_i \cup O_i))$ if $X \in AR$, to prevent the opponent from repeating attacks against the proponent’s arguments that are already attacked. Complexity analysis of the procedure is beyond the scope of this paper. A comparison between our proof procedure and that of Modgil for his semantics [32] remains a future work.

¹⁵We could say that a key question in a semantics for general extended argumentation (including BGW framework) is how the notion of conflict-free should be generalized and what does it mean for an argument to be acceptable? It would be interesting to see how works on logical modes of attacks [27] as well as interpretations in [7] could be applied to provide a formal framework here. This view, first raised in the earlier version of this paper, is also shared by Gabbay in [26]. Note that BGW framework [7] is even more general than the general extended framework studied in this paper as BGW framework allows attacks to come from not only arguments but also attacks.

9 Acknowledgements

We thank the referees for constructive comments and criticisms. This work was partially funded by the Sixth Framework IST program of the European Commission under the 035200 ARGUGRID project.

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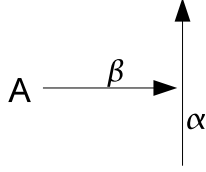
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A Proofs of Lemmas and Theorems in Section 3

Lemma A.1 is needed to prove other Lemmas and Theorems.

Lemma A.1 *Let $S \subseteq AR$ be an i -conflict free set of arguments. There are no attacks $\beta = (A, \alpha), \alpha \in Att$ s.t. $A \in S$ and both β and α are i -defended by S .*

Proof:



Let $S \subseteq AR$ be an i -conflict free set of arguments.

It is sufficient to prove that for every $k \geq 0$, there are no α, β satisfying the following conditions:

1. $\beta = (A, \alpha), A \in S$.
2. S i -defends α within k_α -steps, S i -defends β within k_β -steps, and $k = \min(k_\alpha, k_\beta)$.

We prove by induction on k .

- **Base case:** $k = 0$. Suppose there are α, β satisfying the above conditions with $k = 0$. As A attacks α , α could not be i -defended by S within 0-steps. Hence β is i -defended by S within 0-steps, i.e. there is no attack against β in Att . But as α is i -defended by S , there must exist $B \in S$ s.t. B attacks A and S i -defends (B, A) . Contradiction to the conflict freeness of S .
- **Inductive case:** Suppose there are no α, β satisfying the above conditions with $k = n$.

Assume the contrary that there are α, β satisfying the above conditions with $k = n + 1$.

As α is i -defended by S within k_α -steps, A attacks α , and $A \in S$, there must exist $B \in S$ such that one of the following cases holds:

- B attacks A and S i -defends (B, A) . Contradiction to the conflict freeness of S and this case is not possible.

– B attacks β and S i-defends $\gamma = (B, \beta)$ within $(k_\alpha - 1)$ -steps.

Similarly, as β is i-defended by S within k_β -steps, B attacks β , and $B \in S$, there must exist $C \in S$ such that C attacks γ and S i-defends $\delta = (C, \gamma)$ within $(k_\beta - 1)$ -steps.

Obviously, $n = \min(k_\alpha - 1, k_\beta - 1)$. By the inductive hypothesis, such γ, δ do not exist. Contradiction!

■

A.1 Proof of Lemma 3.1

Let S and S' be sets of arguments s.t. $S \subseteq S'$. We prove by induction on k that if S i-defends an attack β within k -steps then so does S' .

Base case: $k = 0$. Obviously.

Inductive case: Suppose for every $\beta \in Att$, if S i-defends β within k -steps ($k \geq 0$) then so does S' . We need to prove that it also holds for $k + 1$.

Let β be an arbitrary attack that is i-defended by S within $(k+1)$ -steps. There could be two cases:

- S i-defends β within k -steps. By the inductive hypothesis, S' also i-defends β within k -steps.
- For each argument C attacking β there is a $D \in S$ (and hence $D \in S'$) s.t. one of the following cases holds:
 1. D attacks C and S i-defends (D, C) within k -steps. By the inductive hypothesis S' also i-defends (D, C) within k -steps.
 2. D attacks (C, β) and S i-defends $(D, (C, \beta))$ within k -steps. By the inductive hypothesis S' also i-defends $(D, (C, \beta))$ within k -steps.

Therefore β is also i-defended by S' within $(k+1)$ -steps.

A.2 Proof of Lemma 3.2

Let S be an i-admissible set of arguments.

Suppose $A, B \in S$ and A attacks B . From the i-acceptability of B wrt S , there is $C \in S$ such that one the following cases holds:

- C attacks A and S i-defends (C, A) . Contradiction to the conflict freeness of S . This case is not possible.
- C attacks (A, B) and S i-defends $(C, (A, B))$.

So there is $C \in S$ s.t. C attacks (A, B) and S i-defends $(C, (A, B))$.

A.3 Proof of Lemma 3.3

Let S and S' be sets of arguments such that $S \subseteq S'$.

Let A be an argument in $F_I(S)$, i.e. A is i-acceptable wrt S . Let B be an argument attacking A . There must be $C \in S$ (hence $C \in S'$) s.t. one of the following cases holds:

- C attacks (B, A) and S i-defends $(C, (B, A))$. By Lemma 3.1, S' also i-defends $(C, (B, A))$.
- C attacks B and S i-defends (C, B) . By Lemma 3.1, S' also i-defends (C, B) .

As $C \in S'$, A is i-acceptable wrt S' , i.e. $A \in F_I(S')$. Therefore $F_I(S) \subseteq F_I(S')$ and F_I is monotonic.

A.4 Proof of Lemma 3.4

It follows from Lemma A.1 that:

Corollary A.1 *Let S be an i-conflict free set of arguments. Suppose $A \in S$, $B \notin S$, $(A, B) \in \text{Att}$, and S i-defends (A, B) . Then B is not i-acceptable wrt S .*

Proof: Assume the contrary that B is i-acceptable wrt S . There are two cases:

- There is $C \in S$ s.t. C attacks A and S i-defends (C, A) . Contradiction to the i-conflict freeness of S !
- There is $C \in S$ attacking (A, B) and S i-defends $(C, (A, B))$. Contradiction to Lemma A.1.

So B is not i-acceptable wrt S . ■

Let S be an i-admissible set of arguments and A and A' be arguments which are i-acceptable wrt S . Let $S' = S \cup \{A\}$.

1. We prove that S' is i -admissible.

From Lemma 3.3, each argument in S is i -acceptable wrt S' . A is i -acceptable wrt S and so is wrt S' . It means that each argument in S' is i -acceptable to S' . Hence we need only to prove that S' is i -conflict-free.

The following property holds:

Property 1: There are no $C \in S$ and $X \in S'$ s.t. $(C, X) \in Att$ and S i -defends (C, X) .

Proof: Assume the contrary and there are $C \in S$ and $X \in S'$ s.t. $(C, X) \in Att$ and S i -defends (C, X) . There are two cases:

- $X \in S$. Contradiction with the i -conflict freeness of S !
- $X \notin S$. By Corollary A.1, X is not i -acceptable wrt S . Contradiction!

■

Property 2: Suppose $\alpha \in Att$ is i -defended by S within $(k+1)$ steps and there is $X \in S'$ s.t. $\beta = (X, \alpha) \in Att$.

Then there is $C \in S$ s.t. C attacks β and (C, β) is i -defended by S within k -steps.

Proof: As α is i -defended by S within $(k+1)$ -steps, there must exist $C \in S$ s.t. one of the following cases holds:

- C attacks X and S i -defends (C, X) . Contradiction with Property 1! This case is impossible.
- C attacks β and (C, β) is i -defended by S within k -steps.

■

It is sufficient to prove by induction on k that there are no $X, Y \in S'$ s.t. $(X, Y) \in Att$ and S' i -defends (X, Y) within k -steps.

- Base case: $k = 0$. Suppose there are $X, Y \in S'$ s.t. $(X, Y) \in Att$ and S' i -defends (X, Y) within 0-steps, i.e. there is no attack against (X, Y) . From the i -acceptability of Y wrt S , there is $C \in S$ s.t. C attacks X and S i -defends (C, X) . Contradiction to Property 1!

- Inductive case: Suppose there are no $X', Y' \in S'$ s.t. $(X', Y') \in Att$ and S' i-defends (X', Y') within k -steps.

The following property holds.

Property 3: Suppose $\alpha \in Att$ is i-defended by S' within $(m+1)$ -steps, $m \leq k$, and there is $C \in S$ s.t. $\beta = (C, \alpha) \in Att$.

Then there is $X \in S'$ s.t. $\gamma = (X, \beta) \in Att$ and γ is i-defended by S' within m -steps.

Proof: As α is i-defended by S' within $(m+1)$ -steps, there must exist $X \in S'$ s.t. one of the following cases holds:

- X attacks C and S' i-defends (X, C) within m -steps. Contradiction to the inductive hypothesis! This case is impossible.
- X attacks β and $\gamma = (X, \beta)$ is i-defended by S' within m -steps.

■

Suppose there are $X, Y \in S'$ s.t. $\alpha = (X, Y) \in Att$ and S' i-defends α within $(k+1)$ -steps.

From the i-acceptability of Y wrt S , there is $C \in S$ s.t. one of the following cases holds:

- C attacks X and S i-defends (C, X) . Contradiction to Property 1!
- C attacks α and $\beta = (C, \alpha)$ is i-defended by S .

Suppose S i-defends β within m -steps.

By Property 3, there is $X_1 \in S'$ s.t. $\alpha_1 = (X_1, \beta) \in Att$ is i-defended by S' within k -steps.

By Property 2, there is $C_1 \in S$ s.t. $\beta_1 = (C_1, \alpha_1) \in Att$ is i-defended by S within $(m-1)$ -steps.

By Property 3, there is $X_2 \in S'$ s.t. $\alpha_2 = (X_2, \beta_1) \in Att$ is i-defended by S' within $(k-1)$ -steps.

Continue this way and there should be one of the following cases:

- $k \geq m$. There is $C_m \in S$ s.t. $\beta_m = (C_m, \alpha_m) \in Att$ is i-defended by S within 0-steps and α_m is i-defended by S' within $(k-m+1)$ -steps. Contradiction to Property 3!
- $k < m$. There is $X_{k+1} \in S'$ s.t. $\alpha_{k+1} = (X_{k+1}, \beta_k) \in Att$ is i-defended by S' within 0-steps and β_k is i-defended by S within $(m-k)$ -steps. Contradiction to Property 2!

2. As $S \subseteq S'$, from Lemma 3.3, A' is also i -acceptable wrt S' .

A.5 Proof of Theorem 3.1

It is obvious from the fixed point definition of i -complete extensions that each i -preferred extension is an i -complete extension.

Dung ([21]) showed that a complete extension of an AAF is not necessary a preferred extension. So an i -complete extension of an EAF is not necessary an i -preferred extension. ¹⁶

B Proofs of Lemmas and Theorems in Section 4

Lemmas B.1 to B.6 provide the grounds for proving other results.

Lemma B.1 *An argument A is i -acceptable wrt a set $S \subseteq AR$ iff A is g -acceptable wrt $S \cup \Delta(S)$, i.e. $F_I(S) = F_G(S \cup \Delta(S)) \cap AR$.*

Proof:

A is i -acceptable wrt S

\Leftrightarrow for each argument B : if B attacks A then there is argument $C \in S$ s.t. one the following cases holds:

- C attacks (B, A) and $(C, (B, A))$ is i -defended by S , i.e. $(C, (B, A)) \in \Delta(S)$. So $(C, (B, A)) \in S \cup \Delta(S)$.
- C attacks B and (C, B) is i -defended by S , i.e. $(C, B) \in \Delta(S)$. So $(C, B) \in S \cup \Delta(S)$.

$\Leftrightarrow A$ is g -acceptable wrt $S \cup \Delta(S)$. ■

Lemma B.2 *Given an attack $\alpha \in Att$ and a set $S \subseteq AR$ of arguments in a bounded $EAF = (AR, Att)$. From the boundedness of EAF :*

1. α is i -defended by S iff for each argument X attacking α , there is $A \in S$ s.t.

- A attacks X and $(A, X) \in S \cup \Delta(S)$, or

¹⁶Note that each $AAF = (AR, Att)$ could be treated as an $EAF = (AR, Att)$ where $Att = Att^0 \subseteq AR \times AR$.

- A attacks (X, α) and $(A, (X, \alpha)) \in S \cup \Delta(S)$
2. α is g -acceptable wrt $S \cup \Delta(S)$, i.e. $\alpha \in F_G(S \cup \Delta(S))$ iff α is i -defended by S , i.e. $\alpha \in \Delta(S)$.

Proof:

1. if part: it follows directly from the definition of i -defense that: α is i -defended by S if for each argument X attacking α , there is $A \in S$ s.t.
- A attacks X and (A, X) is i -defended by S , i.e. $(A, X) \in \Delta(S) \subseteq S \cup \Delta(S)$, or
 - A attacks (X, α) and $(A, (X, \alpha))$ is i -defended by S , i.e. $(A, (X, \alpha)) \in \Delta(S) \subseteq S \cup \Delta(S)$

Only if part: for an attack $\alpha \in Att$, suppose that for each argument X attacking α , there is $A \in S$ s.t. either cases below occurs.

- A attacks X and $\beta = (A, X)$ is i -defended by S within k_X -steps.
- A attacks (X, α) and $\beta = (A, (X, \alpha))$ is i -defended by S within k_X -steps.

Let $k = \max\{k_X \mid (X, \alpha) \in Att\} + 1$. From the boundedness of the given EAF, k is a finite number. Thus in both cases β is i -defended by S within k -steps. Thus α is i -defended by S .

2. This property follows from property 1 and the definition of g -acceptability: α is g -acceptable wrt $S \cup \Delta(S)$ **iff** for each argument X attacking α , there is $A \in S$ s.t.
- $(A, X) \in S \cup \Delta(S)$, or
 - $(A, (X, \alpha)) \in S \cup \Delta(S)$

■

Lemma B.3 *A set $S \subseteq AR$ is i -conflict free iff $R = S \cup \Delta(S)$ is g -conflict free.*

Proof: This lemma refers to Lemma A.1.

\Rightarrow : Suppose S is i-conflict free. Assume the contrary, that R is not g-conflict free. There exists $A \in S$, $X \in R$ s.t. $(A, X) \in \Delta(S)$. There are two cases:

- $X \in S$. Contradiction to the i-conflict freeness of S !
- $X \in \Delta(S)$. Contradiction to Lemma A.1!

\Leftarrow : Suppose R is g-conflict free. Assume the contrary, that S is not conflict free. There are $A, B \in S$, s.t. (A, B) is i-defended by S , i.e. $(A, B) \in \Delta(S) \subseteq R$. Contradiction to the g-conflict freeness of R !

■

Lemma B.4 *Let $S \subseteq AR$ be a set of arguments and \mathcal{A} be a set of attacks s.t. $\mathcal{A} \subseteq \Delta(S)$ and the set $R = S \cup \mathcal{A}$ is g-admissible. Then*

$$S \cup \Delta(S) \subseteq \bigcup_{k=0}^{\infty} F_G^k(R).$$

Furthermore, the set $\bigcup_{k=0}^{\infty} F_G^k(R)$ is a g-complete extension.

Proof:

From Lemma 4.2, as R is g-admissible and each element in $F_G(R)$ is g-acceptable to R , $R \cup F_G(R)$ is g-admissible. As a consequence, for every k , $R_k = \bigcup_{j=0}^k F_G^j(R)$ is g-admissible.

Let $\Delta_k(S)$ denote the set of attacks in $\Delta(S)$ that are i-defended by S within k -steps.

It is sufficient to prove that for every $k \geq 0$, $\Delta_k(S) \subseteq R_{k+1}$.

We prove by induction on k .

- **Base case:** $k = 0$. Let α be an attack in $\Delta_0(S)$, i.e. α is i-defended by S within 0-steps. There is no argument attacking α , and hence α is g-acceptable wrt R , i.e. $\alpha \in F_G(R) \subseteq R_1$.
- **Inductive case:** Suppose $\Delta_k(S) \subseteq R_{k+1}$. We prove that $\Delta_{k+1}(S) \subseteq R_{k+2}$.

Let α be an attack i-defended by S within $(k+1)$ -steps, i.e. $\alpha \in \Delta_{k+1}(S)$. For each argument C attacking α there is $D \in S$ (and hence $D \in R_{k+1}$) s.t.

- D attacks C and S i-defends (D, C) within k -steps or
- D attacks (C, α) and S i-defends $(D, (C, \alpha))$ within k -steps.

By the inductive hypothesis, all such (D, C) or $(D, (C, \alpha))$ are in R_{k+1} . Therefore α is g -acceptable wrt R_{k+1} , i.e. $\alpha \in F_G(R_{k+1}) \subseteq R_{k+2}$.

So each attack in $\Delta_{k+1}(S)$ is in R_{k+2} , and hence $\Delta_{k+1}(S) \subseteq R_{k+2}$.

Now we prove that the set $\mathcal{E} = \bigcup_{k=0}^{\infty} F_G^k(R) = \bigcup_{k=0}^{\infty} R_k$ is a g -complete extension.

- \mathcal{E} is g -conflict-free.

Assume the contrary. There exist $A, B, (A, B) \in \mathcal{E}$. Thus there exists k_1, k_2, k_3 s.t. $A, B, (A, B)$ are respectively in $R_{k_1}, R_{k_2}, R_{k_3}$. Let $k = \max\{k_1, k_2, k_3\}$. Clearly $A, B, (A, B)$ are all in R_k . So R_k is not g -conflict-free. Contradiction !

- If $\alpha \in \mathcal{E}$, then α is g -acceptable wrt \mathcal{E} , i.e. $\mathcal{E} \subseteq F_G(\mathcal{E})$.

$\alpha \in \mathcal{E}$ implies there is k s.t. $\alpha \in R_k$. Since R_k is g -admissible, α is g -acceptable wrt R_k . Since $R_k \subseteq \mathcal{E}$, α is also g -acceptable wrt \mathcal{E} .

- If $\alpha \in AR \cup Att$ is g -acceptable wrt \mathcal{E} , then $\alpha \in \mathcal{E}$, i.e. $F_G(\mathcal{E}) \subseteq \mathcal{E}$.

α being g -acceptable wrt \mathcal{E} implies: for each argument B attacking α , there exists $C \in \mathcal{E}$ s.t. $(C, B) \in \mathcal{E}$ or $(C, (B, \alpha)) \in \mathcal{E}$, so there exists a number k_B s.t. $(C, B) \in R_{k_B}$ or $(C, (B, \alpha)) \in R_{k_B}$. Since the set of all arguments attacking α is finite (we consider bounded frameworks), the set $\{k_B \mid (B, \alpha) \in Att\}$ is finite. Let m be the maximum of this set. It is clear that α is g -acceptable wrt R_m . Thus $\alpha \in R_{m+1} \subseteq \mathcal{E}$.

■

Lemma B.5 *Let $R \subseteq AR \cup Att$ and $S \subseteq AR$. If R is g -complete and $S \subseteq R$, then $\Delta(S) \subseteq R$.*

Proof: By induction.

For an attack $\beta \in Att$, if β is i-defended by S within 0-step, then it is clear that β is g -acceptable wrt R . As R is g -complete, $\beta \in R$.

Suppose that any attack that is i-defended by S within k -step is in R . Consider an attack β that is i-defended by S within $(k+1)$ -steps. For any argument C attacking β , there exists $D \in S$ s.t. either (1) D attacks C

and S i -defends (D, C) within k -steps, or (2) D attacks (C, β) and S i -defends $(D, (C, \beta))$ within k -steps. By the inductive hypothesis, In case (1), $(D, C) \in R$ and in case (2), $(D, (C, \beta)) \in R$. Thus β is g -acceptable wrt R . As R is g -complete, $\beta \in R$. ■

Lemma B.6 *Let S be an i -admissible set, R be a g -complete extension, and $\Sigma = \bigcup_{k=0}^{\infty} F_I^k(S)$. If $S \subseteq R$ then:*

1. $\Sigma \subseteq R$.
2. Σ is an i -complete extension.
3. $\forall \Sigma' \subseteq AR \cup Att$, if $S \subseteq \Sigma' \subseteq R$ and $F_I(\Sigma') = \Sigma'$ then $\Sigma \subseteq \Sigma'$.

Remark 1 *It follows that, if S is i -admissible subset of the i -grounded extension, then Σ is the i -grounded extension.*

Proof:

Let $S_k = F_I^k(S)$ and by convention: $S_0 = S$. From the i -admissibility of $S_0 \Rightarrow S_0 \subseteq S_1$. Together with the monotonicity of F_I (Lemma 3.3): $S_0 \subseteq S_1 \subseteq \dots \subseteq S_k \subseteq \dots$. From Lemma 3.4, S_k is i -admissible for any k .

Proof of property 1:

By induction, we prove $\forall k : S_k \subseteq R$, as follows. Basic case: $S \subseteq R$ was given. Suppose $S_{i-1} \subseteq R$ for some i . We have $S_i = F_I(S_{i-1}) = F_G(S_{i-1} \cup \Delta(S_{i-1})) \cap AR$ (Lemma B.1). From Lemma B.5, $\Delta(S_{i-1}) \subseteq R \Rightarrow S_{i-1} \cup \Delta(S_{i-1}) \subseteq R \Rightarrow F_G(S_{i-1} \cup \Delta(S_{i-1})) \subseteq F_G(R) = R$. So $S_i \subseteq R$. Thus $\Sigma = \bigcup_{k=0}^{\infty} S_k \subseteq R$.

Proof of property 2: We prove via several observations.

1. *Given $\alpha \in Att$, if $\alpha \in \Delta(\Sigma)$ then $\alpha \in \Delta(S_k)$ for some k .*

We prove by induction on the number of steps within which Σ i -defends α .

Basic case: Σ i -defends α within 0-steps. Clearly $\alpha \in \Delta(S_0)$ since S_0 also i -defends α (within 0-steps).

Hypothesis: If $\alpha \in Att$ is i-defended by Σ within n -steps then $\alpha \in \Delta(S_k)$ for some k .

Consider $\alpha \in Att$ that is i-defended by Σ within $(n + 1)$ -steps. Let $Att_\alpha = \{\beta_1, \dots, \beta_N\}$. Note that Att_α is a finite set (as the framework is bounded).

By definition of i-defense, $\forall j \in \{1, \dots, N\}$, there exists α_j that is i-defended by Σ within n -steps s.t. α_j attacks β_j or $source(\beta_j)$ (note that given $\beta = (A, X)$ then $source(\beta) = A$).

By induction hypothesis, for each α_j , there exists m_j s.t. $\alpha_j \in \Delta(S_{m_j})$. Let $m = \max\{m_1, \dots, m_N\}$. Since $\forall j : S_{m_j} \subseteq S_m$, from Lemma 3.1, $\forall j : \alpha_j \in \Delta(S_m)$.

For each α_j , since $source(\alpha_j) \in \Sigma$, there exists p_j s.t. $source(\alpha_j) \in S_{p_j}$. Let $p = \max\{p_1, \dots, p_N\}$. Clearly $\forall j : source(\alpha_j) \in S_p$.

Now consider $k = \max\{m, p\}$, by Lemma 3.1 $\alpha_j \in \Delta(S_k)$. Moreover S_k contains $source(\alpha_j)$. Thus S_k i-defends α .

2. Σ is i-conflict free: Assume the contrary. There exist $A, B \in \Sigma$ s.t. $(A, B) \in \Delta(\Sigma)$. From $A, B \in \Sigma$, there exist a, b s.t. $A \in S_a$ and $B \in S_b$. Let $c = \max\{a, b\} \Rightarrow A, B \in S_c$. Use observation (1), let $(A, B) \in \Delta(S_d)$. Let $m = \max\{c, d\}$. So $S_c, S_d \subseteq S_m$. Thus $A, B \in S_m$ while $(A, B) \in \Delta(S_m)$ (Lemma 3.1). Thus S_m is not i-conflict free. Contradiction.
3. Σ is i-admissible. Consider $A \in \Sigma$. So $A \in S_a$ for some a . As S_a is i-admissible, $A \in F_I(S_a)$. By the monotonicity of F_I (Lemma 3.3), $A \in F_I(\Sigma)$. Together with observation (2), Σ is i-admissible.
4. Σ is i-complete. Use observation (3), it is sufficient to show that $F_I(\Sigma) \subseteq \Sigma$.

Consider $A \in F_I(\Sigma)$. Let $Att_A = \{B_1, \dots, B_N\}$. Note that Att_A is finite due to the given framework is bounded. Since A is i-acceptable wrt Σ , for each $j \in \{1, \dots, N\}$ there exists $C_j \in \Sigma$ s.t. C_j attacks B_j and Σ i-defends (C_j, B_j) , or C_j attacks (B_j, A) and Σ i-defends $(C_j, (B_j, A))$. Let $Y_j = B_j$ in the former case, and $Y_j = (B_j, A)$ in the latter case. From $C_j \in \Sigma$, there exists m_j s.t. $C_j \in S_{m_j}$. From $(C_j, Y_j) \in \Delta(\Sigma)$ and observation (1), there exists k_j s.t. $(C_j, Y_j) \in \Delta(S_j)$. Let $m = \max\{k_1, \dots, k_N, m_1, \dots, m_N\}$. It follows that $\forall j \in \{1, \dots, N\} : C_j \in S_m$ and S_m i-defends (C_j, Y_j) . Thus $A \in F_I(S_m)$. It follows that $F_I(\Sigma) \subseteq \Sigma$.

Proof of property 3:

Since $S_0 \subseteq \Sigma'$, $S_1 \subseteq F_I(\Sigma') = \Sigma'$, and by induction $S_k \subseteq \Sigma'$. Hence $\Sigma \subseteq \Sigma'$.

■

B.1 Proof of Lemma 4.1

Let R and R' be sets of arguments/attacks such that $R \subseteq R'$.

Let X be an argument/attack in $F_G(R)$, i.e. X is g-acceptable wrt R . Let A be an argument attacking X . There must be $B \in R$ (hence $B \in R'$) s.t. one of the following cases holds:

- $(B, (A, X)) \in R$. So $(B, (A, X)) \in R'$.
- $(B, A) \in R$. So $(B, A) \in R'$.

Hence X is i-acceptable wrt R' , i.e. $A \in F_G(R')$. Therefore $F_G(R) \subseteq F_G(R')$ and F_G is monotonic.

B.2 Proof of Lemma 4.2

Let $R \subseteq AR \cup Att$ be a g-admissible set. Let $X, X' \in AR \cup Att$ s.t. X, X' are g-acceptable wrt R . Let $R' = R \cup \{X\}$.

1. We prove that R' is g-admissible.

If $X \in R$ then $R' = R$ and R' is g-admissible.

Suppose $X \notin R$. From the monotonicity of F_G (Lemma 4.1), X and all elements in R are g-acceptable wrt R' . So we need only to prove that R' is g-conflict free.

Assume the contrary. There must be A, β such that $A, (A, \beta), \beta$ are all in R' . Since R is g-conflict free, one of the following cases holds:

- $A = X$. Hence $(A, \beta) \in R$. There are two cases:
 - $\beta \neq A$. Hence $\beta \in R$. Because β is g-acceptable wrt R there is $C \in R$ s.t. either $(C, (A, \beta)) \in R$ or $(C, A) \in R$.
If $(C, (A, \beta)) \in R$, then $C, (C, (A, \beta)), (A, \beta)$ are all in R , contradicting the g-conflict freeness of R .

If $(C, A) \in R$, then from the g-acceptability of $A(=X)$ wrt R , there is $D \in R$ s.t. $(D, C) \in R$ or $(D, (C, A)) \in R$.

In the first case, $D, (D, C), C \in R$.

In the second case, $D, (D, (C, A)), (C, A) \in R$.

Thus both cases contradict with the g-conflict freeness of R .

- $\beta = A$ (i.e. A self-attacks). Hence $(A, A) \in R$. It follows from the g-acceptability of A wrt R that there is $C \in R$ s.t. (a) $\alpha = (C, A) \in R$ or (b) $\alpha' = (C, (A, A)) \in R$.

If (a) occurs: Because A is g-acceptable wrt R , there is $D \in R$ s.t. $(D, \alpha) \in R$ or $(D, C) \in R$. Hence $D, (D, \alpha), \alpha$ are all in R or $D, (D, C), C$ are all in R . Contradiction with the g-conflict freeness of R .

If (b) occurs: $C, (C, (A, A))$ and (A, A) are all in R . Contradiction with the g-conflict freeness of R .

Therefore this case is not possible.

- $X = \beta$. Hence $(A, \beta) \in R$.

As proved before, it is not possible that $A = \beta$.

Let $\beta \neq A$. Hence $A \in R$. Because β is g-acceptable wrt R , there is $C \in R$ s.t. $(C, A) \in R$ or $(C, (A, \beta)) \in R$. Hence R is not g-conflict free. Contradiction!

- $X = (A, \beta)$. Hence $A, \beta \in R$. Because β is g-acceptable wrt R , there is $C \in R$ s.t. $(C, A) \in R$ or $(C, (A, \beta)) \in R$. Because R is g-conflict free, it follows that $(C, (A, \beta)) \in R$.

Because (A, β) is g-acceptable wrt R , there is $D \in R$ s.t. $(D, C) \in R$ or $(D, (C, (A, \beta))) \in R$. Hence R is not g-conflict free. Contradiction!

2. As $R \subseteq R'$, from Lemma 4.1, X' is also g-acceptable wrt R' .

B.3 Proof of Lemma 4.3

Let $S \subseteq AR$ be a set of arguments. We prove that S is i-admissible iff $R = S \cup \Delta(S)$ is g-admissible.

\Rightarrow : Suppose S is i-admissible. By Lemma B.3, as S is i-conflict free, R is g-conflict free. Let X be an element in R . There are two cases:

- $X \in S$. By Lemma B.1, X is g-acceptable wrt R .
- $X \in \Delta(S)$. By Lemma B.2, X is g-acceptable wrt R .

Hence X is g-acceptable wrt R and R is g-admissible.

\Leftarrow : Suppose R is g-admissible. By Lemma B.3, as R is g-conflict free, S is i-conflict free.

Let A be an argument in S . Let B be an argument attacking A . From the g-acceptability of A wrt R , there is $C \in R \cap AR = S$ s.t.

- $(C, B) \in R$, i.e. $(C, B) \in \Delta(S)$, or
- $(C, (B, A)) \in R$, i.e. $(C, (B, A)) \in \Delta(S)$.

Hence A is i-acceptable wrt S . Hence S is i-admissible.

B.4 Proof of Theorem 4.1

Let $S \subseteq AR$ be a set of arguments. We prove that S is an i-complete extension iff $R = S \cup \Delta(S)$ is a g-complete extension.

By Lemma 4.3, S is i-admissible iff R is g-admissible.

Suppose S is an i-complete extension. Let $X \in AR \cup Att$ s.t. X is g-acceptable wrt $S \cup \Delta(S)$:

- If $X \in AR$ then X is i-acceptable wrt S (By Lemma B.1) and hence $X \in S$ and so $X \in R$.
- If $X \in Att$ then X is i-defended by S (By Lemma B.2) and hence $X \in \Delta(S)$ and so $X \in R$.

It means that each argument/attack g-acceptable wrt R belongs to R and hence R is a g-complete extension.

Suppose $S \cup \Delta(S)$ is a g-complete extension. Let $A \in AR$ s.t. A is i-acceptable wrt S . By Lemma B.1, A is g-acceptable to S . Hence $A \in S$. It means that each argument i-acceptable wrt S belongs to S and hence S is an i-complete extension.

B.5 Proof of Theorem 4.2

Let $S_k = F_I^k(\emptyset)$. First we prove by induction that given a g-complete extension R , for any k : $S_k \subseteq R$.

Basic case: $S_0 = \emptyset \subseteq R$. Suppose that for some $n \geq 0$: $S_n \subseteq R$. From Lemma B.1, $F_I(S_n) = F_G(S_n \cup \Delta(S_n)) \cap AR$. From Lemma B.5, $\Delta(S_n) \subseteq R$, hence $S_n \cup \Delta(S_n) \subseteq R$. From the monotonicity of F_G (Lemma 3.3) and R being g-complete, $F_G(S_n \cup \Delta(S_n)) \subseteq F_G(R) = R$. Thus $S_{n+1} \subseteq R$. Hence $\forall k : S_k \subseteq R$.

Now consider $S = \bigcup_{k=0}^{\infty} S_k$. Because $S \subseteq R$, apply Lemma B.5 $\Rightarrow S \cup \Delta(S) \subseteq R$.

Thus if $S \cup \Delta(S)$ is a g-complete extension, then it is also the g-grounded extension. By theorem 4.1, if S is the i-grounded extension then $S \cup \Delta(S)$ is g-complete.

Hence it remains to show that S is the i-grounded extension. This follows from Lemma B.6 as $S_0 = \emptyset$ is i-admissible.

B.6 Proof of Theorem 4.3

The following lemmas hold.

Lemma B.7 *Suppose S_1, S_2 are i-admissible and subsets of a g-complete extension R . Then $S = S_1 \cup S_2$ is i-admissible.*

Proof: First we prove that S is conflict free. Assume the contrary. There are $A, B \in S$ s.t. $(A, B) \in Att$ and S i-defends (A, B) . Because $S \subseteq R_0 = R \cap AR$, R_0 i-defends (A, B) . From Lemma B.5, $\Delta(R_0) \subseteq R$. Hence $(A, B) \in R$. So $A, (A, B), B$ are all in R . Contradiction with the g-conflict freeness of R .

Consider $A \in S$. Hence A in S_1 or $S_2 \Rightarrow A$ is i-acceptable wrt S_1 or $S_2 \Rightarrow A$ is i-acceptable wrt S (by Lemma 4.1). Hence S is i-admissible. ■

It follows immediately from Lemma B.7 that:

Lemma B.8 *Let Σ be the union of all i-admissible sets that are subsets of a g-complete extension R . Then Σ is i-admissible.*

From Lemma B.5 it follows immediately:

Lemma B.9 *Suppose S is an i-admissible set and a subset of a g-complete extension R . Then $F_I(S) \cup \Delta(S) \subseteq R$.*

Now we prove theorem 4.3

Let R be a g-complete extension. Let Σ be the union of all i-admissible subsets of R . Obviously $\Sigma \subseteq R$. Σ is i-admissible (By Lemma B.8). As $\Sigma \subseteq R$, $F_I(\Sigma) \subseteq R$ (By Lemma B.9) and i-admissible. It follows that $F_I(\Sigma) \subseteq \Sigma$. As Σ is i-admissible, $F_I(\Sigma) \supseteq \Sigma$. So $F_I(\Sigma) = \Sigma$ and Σ is an i-complete extension.

Σ is also a greatest (wrt set inclusion) i-admissible subset of R and hence Σ is also a greatest (wrt set inclusion) i-complete extension that is a subset of R .

C Proofs of Lemmas and Theorems in Section 5

C.1 Proof of Theorem 5.1

Before giving the proof, we introduce an useful notion: for $S \subseteq AR$, $\Pi(S) = \{\alpha \in \Delta(S) \mid source(\alpha) \in S\}$.

1. $A \in SP_0$. Since in step 3.a (definition 5.1), $SP_i = SP_{i+1}$ while in step 3.b, $SP_i \subsetneq SP_{i+1}$, $SP_0 \subseteq SP_1 \cdots \subseteq SP_n$. Thus $A \in SP_n$.

Let \mathcal{P} denote $\bigcup_{i=0}^n P_i$. So $\mathcal{P} = SP_n \cup \mathcal{A}$ with $\mathcal{A} = \mathcal{P} \cap Att$. To prove that SP_n is i -admissible, we first prove several observations below.

- (a) For any attack $\alpha \in \mathcal{A}$: $\alpha \in \Pi(SP_n)$.

Let i_1, \dots, i_k be the sequence of all steps at which an attack α_j ($1 \leq j \leq k$) is selected from P_{i_j} . It follows that α_k is the last attack selected from some P_i and for all $m > i_k$, $P_m \subseteq AR$.

We prove by backward induction (from α_k down to α_1) that all of them are i -defended by SP_n .

- Basic case: Clearly $Attack_{\alpha_k} = \emptyset$, since $Attack_{\alpha_k} \neq \emptyset$ would give rise to elements of $Attack_{\alpha_k}$ to be added to O component by step 3.a. And later when these elements are selected by step 3.b, attacks against them are added to P -component, making α_k not the last attack selected from P -component any more. Thus α_k is i -defended by SP_n within 0-steps.
- Suppose $\alpha_{l+1}, \dots, \alpha_k$ are all i -defended by SP_n . We prove that so is α_l . Consider step i_l at which α_l is selected from P_{i_l} by step 3.a, elements of $Attack_{\alpha_l}$ are added to O_{i_l} . Each of them is selected later by step 3.b, resulting in elements $D, (D, Y)$ added to P -component. This establishes that any attack against α_l is counter-attacked by an argument $D \in SP_n$ with an attack $(D, Y) \in \{\alpha_{l+1}, \dots, \alpha_k\}$. By the induction hypothesis, (D, Y) is i -defended by SP_n . Thus $\alpha_l \in \Delta(SP_n)$.

- (b) Any argument $C \in SP_n$ is i -acceptable wrt SP_n .

Clearly when C is selected at step i by 3.a, all elements of $Attack_C$ are added to O -component. When such an element $(B, C) \in Attack_C$ is selected at some later step by 3.b, $D, (D, Y)$ representing an attack against B or (B, C) are added to P -component.

Since $D \in SP_n$ and $(D, Y) \in \Pi(SP_n)$ by observation (a), C is i -acceptable wrt SP_n .

(c) *If α is added to P_i at a step i and $\alpha \in O_j$ for some j , then $i < j$.*
This result follows immediately from step 3.b: for any $\alpha \in Att$ added to P_i , $\alpha \notin (SO_i \cup O_i)$.

(d) *If $\alpha \in \mathcal{A}$, then $source(\alpha) \in SP_n$.* This follows from step 3.b.

(e) $\mathcal{A} \cap SO_n = \emptyset$

Assume $DS = \mathcal{A} \cap SO_n \neq \emptyset$. For $\alpha \in DS$, let $eject(\alpha) = \max\{i \mid \alpha \in O_i\}$, i.e. α is selected from $O_{eject(\alpha)}$ and is not added to any O_j for $j > i$. And let $\alpha_0 = (B, \beta) \in DS$ s.t. $eject(\alpha_0) = \max\{eject(\alpha) \mid \alpha \in DS\}$. Because $\alpha_0 \in \mathcal{A}$, it follows that $B \in SP_n$ (observation (d)).

Consider the step $eject(\alpha_0)$ at which α_0 is selected from $O_{eject(\alpha_0)}$ by step 3b, resulting in an attack α_1 being added to $P_{eject(\alpha_0)}$. There are two cases:

- i. $\alpha_1 = (D, \alpha_0)$. Since $\alpha_0 \in DS$ and hence $\alpha_0 \in \mathcal{P}$, $\alpha_1 \in O_m$ at some step m . Hence $\alpha_1 \in DS$. From observation (c), $eject(\alpha_0) < m$. It follows that $eject(\alpha_0) < eject(\alpha_1)$. Contradiction.
- ii. $\alpha_1 = (D, B)$. The first bullet of step 3b says $B \notin SP_{eject(\alpha_0)}$. So $B \notin P_{eject(\alpha_0)}$. But since $B \in SP_n$, B must be added to P -component at some step after $eject(\alpha_0)$, so gets selected at some step $j > eject(\alpha_0)$. Here, $\alpha_1 \in DS$ due to $\alpha_1 \in O_{j+1}$ and $\alpha_1 \in P_{eject(\alpha_0)+1}$. However $eject(\alpha_1) > j > eject(\alpha_0)$. Contradiction.

(f) \mathcal{P} is g -admissible.

i. \mathcal{P} is g -conflict-free.

Assume the contrary. There exist $A, (A, \beta), \beta \in \mathcal{P}$. When β is selected from some P_i , (A, β) is added to O_i . Thus $(A, \beta) \in SO_n$. This is impossible by observation (e).

ii. *For any $\beta \in \mathcal{P}$, β is g -acceptable wrt \mathcal{P} .*

Since β is selected from P_i at some step i , for any attack $(B, \beta) \in Att$, $(B, \beta) \in O_{i+1}$ by step 3.a. Since $O_n = \emptyset$, any such attack (B, β) is selected later at some step j by step 3.b, giving rise to (D, Y) and (possibly) D (if $D \notin SP_j$) being added to P_j . From the fact that $Y = B$ or $Y = (B, \beta)$ and $(D, Y) \in \mathcal{P}$, β is g -acceptable wrt \mathcal{P} .

(g) SP_n is i-conflict-free.

Applying Lemma B.4 for the set \mathcal{P} , we have $SP_n \cup \Delta(SP_n) \subseteq \bigcup_{k=0}^{\infty} F_G^k(\mathcal{P})$. Since by the same lemma $\bigcup_{k=0}^{\infty} F_G^k(\mathcal{P})$ is g-admissible, $SP_n \cup \Delta(SP_n)$ is g-conflict-free. By Lemma B.3, SP_n is i-conflict-free.

From observations (b),(g) SP_n is i-admissible.

2. Let A be an argument of an i-admissible set S . Given a selection sl , we build a dispute derivation for A . At each step i , the constructed tuple $t_i = \langle P_i, O_i, SP_i, SO_i \rangle$ satisfies the following properties:

(a) $P_i \subseteq S \cup \Pi(S)$ and $SP_i \subseteq S$

(b) O_i and SO_i consist of attacks $(B, \alpha) \in Att$ with $\alpha \in S \cup \bigcup_{j=0}^{i-1} P_j$.

The first tuple $t_0 = \langle \{A\}, \emptyset, \{A\}, \emptyset \rangle$ clearly satisfies both properties. Suppose we have constructed t_i . Let X be the element selected by sl at step i . We can construct t_{i+1} as follows:

- Case $X \in P_i$: Clearly t_{i+1} can be constructed by step 3.a. Elements added to O_{i+1} are from $Attack_X$ with $X \in P_i$, hence both properties (a),(b) hold for t_{i+1} .
- Case $X = (B, \alpha) \in O_i$: Hence, $\alpha \in P_j$ for some $j < i$. Because of i-admissibility of S , and $P_j \subseteq S \cup \Pi(S)$, it follows there is $D \in S$ s.t. (1) D attacks (B, α) and $(D, (B, \alpha))$ is i-defended by S or (2) D attacks B and (D, B) is i-defended by S . Let $Y = (B, \alpha)$ or $Y = B$ in cases (1) or (2). Note that due to the i-conflict-freeness of S , (2) does not happen if $B \in SP_i \subseteq S$. Further, if $\alpha \in \Pi(S)$, then D is selected such that $rank((D, Y)) < rank(\alpha)$, where $rank(\gamma)$ for some $\gamma \in \Pi(S)$ is a number s.t. S i-defends γ within $rank(\gamma)$ steps but not within $rank(\gamma) - 1$ steps. This selection of D is always possible due to Definition 3.1 of i-defense. We will show shortly below that the constraint $rank((D, Y)) < rank(\alpha)$ induces a lexicographical order on tuples t_0, t_1, t_2, \dots of the constructed dispute derivation, making the derivation finite.

But first we need to prove that t_{i+1} satisfies both properties (a) and (b). To show $(D, Y) \notin SO_i \cup O_i$, assume the contrary. From $(D, Y) \in SO_i \cup O_i$, it follows that $Y \in P_j$ for some $j < i$. From the

property (a), $Y \in S \cup \Pi(S)$. $Y \in S$ contradicts with the conflict-freeness of S since $D, Y \in S$ while $(D, Y) \in \Pi(S)$. $Y \in \Pi(S)$ contradicts with the basic Lemma A.1 since (D, Y) is an attack on Y while both $(D, Y), Y \in \Pi(S)$. Hence we can construct t_{i+1} from t_i by adding (D, Y) and (possibly) D (if $D \notin SP_i$) to P_i to obtain P_{i+1} . It is easy now to see that t_{i+1} satisfies both properties (a) and (b).

It remains to show that the constructed dispute derivation is finite. Suppose it is infinite. Because an argument will not be re-defended, there is a constant K s.t. for all $k > K$ only attacks are selected from P_k .

Given P_k with $k > K$, let $\mathcal{A}(P_k)$ denote a (finite) partition $[A_0, A_1, \dots, A_i, \dots]$ on $P_k \cap Att$ such that for $\alpha \in A_i$, $rank(\alpha) = i$. Clearly if an attack is selected from P_k , then it is an element of some A_i of this partition.

For $\beta = (B, \alpha) \in O_k$, let $rank(\beta) = rank(\alpha)$ if $\alpha \in \Pi(S)$, and $rank(\beta) = \perp$ otherwise. Because for all $k > K$ only attacks are selected from P_k , for any attack β added to O -component at steps after K : $rank(\beta) \neq \perp$. Moreover since O_K has a finite number of attacks β s.t. $rank(\beta) = \perp$, there exists a constant $M > K$ s.t. for any attack β selected from O_m at a step $m > M$: $rank(\beta) \neq \perp$.

Now, given O_m with $m > M$, let $\mathcal{B}(O_m)$ denote a (finite) partition $[B_1, \dots, B_j, \dots]$ on $\{\beta \in O_m \mid rank(\beta) \neq \perp\}$, where $B_j = \{\beta \mid rank(\beta) = j\}$. Clearly if an attack is selected from O_m , then it is an element of some B_j in this partition.

For a tuple $t = \langle P, O, SP, SO \rangle \in \{t_m \mid m > M\}$, let $\delta(t)$ denote the sequence $A_0 B_1 A_1 B_2 \dots B_l A_l$ where $[A_0, A_1, \dots, A_l] = \mathcal{A}(P)$ and $[B_1, \dots, B_l] = \mathcal{B}(O)$ (note that \emptyset can be padded at the ends of two partitions to make them of equal length). Now we define a lexicographical order \sqsupset on the set $\{t_m \mid m > M\}$, as follows.

Given $t, t' \in \{t_m \mid m > M\}$, let $\delta(t) = A_0 B_1 A_1 \dots B_q A_q \dots B_l A_l$ and $\delta(t') = A'_0 B'_1 A'_1 \dots B'_q A'_q \dots B'_l A'_l$. We define $t \sqsupset t'$ iff there exists a number q such that either conditions below hold.

- $B'_q \subsetneq B_q$ and $A_q B_{q+1} \dots B_l A_l = A'_q B'_{q+1} \dots B'_l A'_l$
- $A'_q \subsetneq A_q$ and $B_{q+1} A_{q+2} \dots B_l A_l = B'_{q+1} A'_{q+2} \dots B'_l A'_l$

That for $i > M$, $t_i \sqsupset t_{i+1}$ can be seen from the following observations:

- If at step i , α is selected from P_i , then $\alpha \in A_q$ for some $A_q \in \delta(t_i)$ with $q = \text{rank}(\alpha)$. Further, for each attack β added to O_i to obtain O_{i+1} : $\text{rank}(\beta) = \text{rank}(\alpha)$
- If at step i , α is selected from O_i , then $\alpha \in B_q$ for some $B_q \in \delta(t_i)$ with $q = \text{rank}(\alpha)$. Further, the attack β added to P_i to obtain P_{i+1} satisfies: $\text{rank}(\beta) < \text{rank}(\alpha)$.

Since we assume the process is infinite, we have an infinite sequence $t_M \sqsupset t_{M+1} \sqsupset \dots$. Thus the set $\{t \mid t_M \sqsupset t\}$ is infinite. But since any element of $\delta(t_M)$ is a subset of Att which is finite, there is a finite number of tuples t such that $t_M \sqsupset t$. Contradiction.

D Proofs of Lemmas and Theorems in Section 6

D.1 Proof of Lemma 6.1

1. Suppose both (A, α) and A are g-acceptable wrt R . Consider $\beta \in \text{Att}$ that defeats (A, α) . There are two cases:
 - β defeats (A, α) directly, i.e. $\beta = (B, (A, \alpha))$ for some $B \in AR$. Now since (A, α) is g-acceptable wrt R , there exists $C \in R$ s.t.:
 - $(C, \beta) \in R$, hence (C, β) defeats β directly, or
 - $(C, B) \in R$, hence (C, B) defeats β indirectly.
 - β defeats (A, α) indirectly, i.e. $\beta = (B, A)$ for some $B \in AR$. Now since A is g-acceptable wrt R , there exists $C \in R$ s.t.:
 - $(C, \beta) \in R$, hence (C, β) defeats β directly, or
 - $(C, B) \in R$, hence (C, B) defeats β indirectly.

So there is always some attack of R that defeats β . Hence α is bcgg-acceptable wrt R .

2. Suppose $B \in AR$ is g-acceptable wrt R . Consider an attack $\alpha = (C, B)$ that defeats B (directly). So C attacks B and from the g-acceptability of B wrt R , there exists $D \in R$ s.t. either $(D, C) \in R$ or $(D, \alpha) \in R$, i.e. α is defeated either by (D, C) or by (D, α) . Hence B is also bcgg-acceptable wrt R .

D.2 Proof of Lemma 6.2

1. *If $\alpha \in R$ then $source(\alpha) \in R$.*

Consider an attack β that defeats $source(\alpha)$, and hence defeats α indirectly. Since α is bcgg-acceptable wrt R there exists an attack $\gamma \in R$ s.t. γ defeats β . Thus $source(\alpha)$ is also bcgg-acceptable wrt R . Since R is bcgg-complete, $source(\alpha) \in R$.

2. *$F_G(R) \setminus R \subseteq Att$. It is sufficient to show that for any argument $A \in AR$ that is g-acceptable wrt R : $A \in R$.*

Consider argument B attacking A , there exists $C \in R$ s.t. either $(C, B) \in R$ (i.e. (C, B) defeats (B, A) indirectly) or $(C, (B, A)) \in R$ (i.e. $(C, (B, A))$ defeats (B, A) directly). Hence A is bcgg-acceptable wrt R . Since R is bcgg-acceptable complete, $A \in R$.

3. *If $\alpha \in F_G(R) \setminus R$ then $source(\alpha) \notin R$. Assume the contrary that $source(\alpha) \in R$. We show a contradiction that $\alpha \in R$.*

Consider an attack β that defeats α . Either of the following cases occur:

- β defeats α directly. Since α is g-acceptable wrt R , there exists $C \in R$ s.t. $(C, \beta) \in R$ or $(C, source(\beta)) \in R$, i.e. β is defeated directly by (C, β) or indirectly by $(C, source(\beta))$. Hence α is bcgg-acceptable wrt R . Since R is bcgg-complete, $\alpha \in R$.
- β defeats α indirectly. So β defeats $source(A)$ directly. Since $source(A) \in R$ while R is bcgg-complete, $source(A)$ is bcgg-acceptable wrt R . Thus there exists $\sigma \in R$ that defeats β . So α is bcgg-acceptable wrt R . Hence $\alpha \in R$.

Lemma D.1 below will be used to prove Theorems 6.1, 6.2, and 6.3.

Lemma D.1 *Let $R \subseteq AR \cup Att$. If R is a bcgg-complete extension, then $R \subseteq F_G(R)$.*

Proof: For $X \in R$, consider an argument A attacking X , i.e. $(A, X) \in Att$. Since X is bcgg-acceptable wrt R , there exists an attack $\alpha \in R$ that defeats (A, X) . By Lemma 6.2 property 1: $source(\alpha) \in R$. Hence X is also g-acceptable wrt R . Thus $R \subseteq F_G(R)$. ■

D.3 Proof of Theorem 6.1

1. If R is g-complete, then $rd(R)$ is bcfg-complete because:

(a) $rd(R)$ is bcfg-conflict-free. Assume the contrary. Either of the following cases can occur:

- There exist $(A, \alpha), \alpha \in rd(R)$, i.e. (A, α) directly defeats α . $(A, \alpha) \in rd(R)$ implies $A \in R$. So R is not g-conflict-free as $A, (A, \alpha), \alpha$ are all in R . Contradiction !
- There exist $(A, B), (B, \beta) \in rd(R)$, i.e. (A, B) indirectly defeat (B, β) . $(A, B) \in rd(R)$ implies $A \in R$; $(B, \beta) \in rd(R)$ implies $B \in R$. So R is not g-conflict-free as $A, (A, B), B$ are all in R .

(b) *Each $X \in rd(R)$ is bcfg-acceptable wrt $rd(R)$.*

Consider $\alpha \in Att$ that defeats $X \in rd(R)$.

- If α defeats X directly, then $\alpha = (A, X)$ for some argument $A \in AR$. Since X is also in R , X is g-acceptable wrt R . Thus there exists $B \in R$ s.t. either cases below occurs:
 - $(B, \alpha) \in R$. Hence $(B, \alpha) \in rd(R)$ defeats α directly.
 - $(B, A) \in R$. Hence $(B, A) \in rd(R)$ defeats α indirectly.
- If α defeats X indirectly, then $\alpha = (A, B)$ and $X = (B, \beta)$ for some $A, B \in AR$ and $\beta \in AR \cup Att$. $(B, \beta) \in rd(R)$ implies $B \in R$. Hence B is also g-acceptable wrt R . Since A attacks B , there exists $C \in R$ s.t. either cases below occurs.
 - $(C, \alpha) \in R$. Hence $(C, \alpha) \in rd(R)$ defeats α directly.
 - $(C, A) \in R$. Hence $(C, A) \in rd(R)$ defeats α indirectly.

So there is always an attack of $rd(R)$ that defeats α . Thus X is bcfg-acceptable wrt $rd(R)$.

(c) *If X is bcfg-acceptable wrt $rd(R)$, then $X \in rd(R)$.*

Suppose X is bcfg-acceptable wrt $rd(R)$. Consider two cases below:

- $X \in AR$: Consider an attack $\alpha = (A, X)$ that defeats X . There exists $\beta \in rd(R)$ that defeats α . Since $source(\beta)$, the source argument of β , is in R , X is g-acceptable wrt R . As R is g-complete, $X \in R$. So $X \in rd(R)$.
- $X \in Att$: We show that X is g-acceptable wrt R . Let $(B, X) \in Att$, i.e. (B, X) defeats X directly. There exists $\beta \in rd(R)$ that defeats (B, X) . Since $source(\beta) \in R$, X is also g-acceptable wrt R . As R is g-complete, $X \in R$.

Now we show that $source(X) \in R$. Consider an argument C attacking $source(X)$. Clearly $(C, source(X))$ defeats X indirectly, thus there exists an attack $\gamma \in rd(R)$ that defeats $(C, source(X))$. As $source(\gamma) \in R$, $source(X)$ is g-acceptable wrt R . As R is g-complete, $source(X) \in R$.

Hence $X \in rd(R)$.

2. If R is bcfg-complete, then $F_G(R)$ is g-complete because:

- (a) $F_G(R)$ is g-conflict-free. Assume the contrary. There exist elements $A, (A, X), X \in F_G(R)$. Since X is g-acceptable wrt R while A attacks X , there exists $B \in R$ such that either cases below occur:
- $(B, A) \in R$. Now A is g-acceptable wrt R but is attacked by B , hence there exists $C \in R$ satisfying either conditions below, both of which contradict with the bcfg-conflict-freeness of R .
 - $(C, B) \in R$. Here (C, B) defeats (B, A) indirectly while both of them are in R .
 - $(C, (B, A)) \in R$. Here $(C, (B, A))$ defeats (B, A) directly while both of them are in R .
 - $(B, (A, X)) \in R$. Now (A, X) is g-acceptable wrt R but is attacked by B , hence there exists $C \in R$ satisfying either conditions below, both of which contradict with the bcfg-conflict-freeness of R .
 - $(C, B) \in R$. Here (C, B) defeats $(B, (A, X))$ indirectly while both of them are in R .
 - $(C, (B, (A, X))) \in R$. Here $(C, (B, (A, X)))$ defeats $(B, (A, X))$ directly while both of them are in R .

Thus $F_G(R)$ is g-conflict-free.

(b) $R \subseteq F_G(R)$ by Lemma D.1.

(c) $F_G(R) = F_G^2(R)$. Consider $X \in F_G^2(R)$ and $(A, X) \in Att$. There exists $B \in F_G(R)$ s.t. $(B, Y) \in F_G(R)$ for either $Y = A$ or $Y = (A, X)$. For reasons below X is g-acceptable wrt R :

- $(B, Y) \in R$: From Lemma 6.1 property 1: (B, Y) is bcfg-acceptable wrt R . As R is bcfg-complete, $(B, Y) \in R$.
- $B \in R$: From $(B, Y) \in R$ and Lemma 6.2 property 1 $\Rightarrow source((B, Y)) \in R \Rightarrow B \in R$.

Hence $X \in F_G(R) \Rightarrow F_G^2(R) \subseteq F_G(R)$. Together with $R \subseteq F_G(R) \Rightarrow F_G(R) = F_G^2(R)$.

D.4 Proof of Theorem 6.2

1. Let S be an i-complete extension. From Theorem 4.1, $S \cup \Delta(S)$ is g-complete. From Theorem 6.1, $rd(S \cup \Delta(S)) = S \cup \Pi(S)$ is bcgg-complete.
2. Let S be a set of arguments s.t. $S \cup \Pi(S)$ is a bcgg-complete extension. By Lemma D.1, $S \subseteq F_G(S \cup \Pi(S))$. By Lemma 6.2 property 2: $F_G(S \cup \Pi(S)) \setminus (S \cup \Pi(S)) \subseteq Att$. Hence we can write $F_G(S \cup \Pi(S)) = S \cup \Delta$ where $\Delta \subseteq Att$. By theorem 6.1, $S \cup \Delta$ is g-complete. From Lemma B.5, $\Delta(S) \subseteq \Delta$.

For $\beta \in \Delta$, consider argument B attacking β . Since β is g-acceptable wrt $S \cup \Pi(S)$, there exists $C \in S$ s.t. either $(C, B) \in \Pi(S)$ or $(C, (B, \beta)) \in \Pi(S)$. Together with the boundedness of the considered EAF, $S \cup \Pi(S)$ i-defends β . So $\beta \in \Delta(S)$.

Thus $S \cup \Delta = S \cup \Delta(S)$ is g-complete. By Theorem 4.1, S is i-complete.

D.5 Proof of Theorem 6.3

Let R be a bcgg-complete extension. By theorem 6.1, $F_G(R)$ is a g-complete extension. Let S be the greatest i-complete extension that $F_G(R)$ contains according to theorem 4.3. We show that S is also a greatest i-complete extension that R contains. From Lemma 6.2 property 2: $F_G(R) \setminus R \subseteq Att$, it follows that $S \subseteq R$. Now it remains to prove that there is no i-complete subset S' of R s.t. $S \subsetneq S'$. Assume the contrary. We would have $S \subsetneq S' \subseteq R \subseteq F_G(R)$ (Lemma D.1). Contradiction with the selection of S .

D.6 Proof of Lemma 6.3

We first prove:

Lemma D.2 *Let S be a m -conflict free set of arguments. Suppose $X \in S$ attacks $Y \in AR$ and S i-defends (X, Y) . Then $X \rightarrow^S Y$, i.e. there is no $Z \in S$ attacking (X, Y) .*

Proof: Suppose the contrary and $Y_1 \in S$ is an argument attacking (X, Y) . As S i-defends (X, Y) , there must exist $X_1 \in S$, such that X_1 attacks Y_1 and (X_1, Y_1) is i-defended by S . Let $k \geq 0$ be a number s.t. (X, Y) is

i -defended by S within k -steps but (X, Y) is not i -defended by S within $(k-1)$ -steps. Then (X_1, Y_1) is i -defended by S within $(k-1)$ -steps but (X_1, Y_1) is not i -defended by S within $(k-2)$ -steps.

As S is m -conflict free, $(Y_1, X_1) \notin Att$ and there is $Y_2 \in S$, s.t. Y_2 attacks (X_1, Y_1) . As $(X_1, Y_1) \in \Delta(S)$, there must exist $X_2 \in S$ s.t. $(X_2, Y_2) \in \Delta(S)$. (X_2, Y_2) is i -defended by S within $(k-2)$ -steps but (X_2, Y_2) is not i -defended by S within $(k-3)$ -steps.

Continue this way and there are $X_k, Y_k \in S$, (X_k, Y_k) is i -defended by S within 0 -steps, i.e. there is no $C \in S$ attacking (X_k, Y_k) . Contradiction to the m -conflict freeness of S ! ■

From Lemma D.2, it follows:

Lemma D.3 *In a bounded EAF, suppose that a m -conflict free set of arguments S i -defends an attack (A, B) and $A \in S$. Then $A \rightarrow^S B$ and there is a reinstatement set for $A \rightarrow^S B$.*

Proof: We prove by induction that if S i -defends (A, B) within k -steps, $k \geq 0$, then there is a reinstatement set for $A \rightarrow^S B$.

- **Base case:** $k = 0$. There is no argument attacking (A, B) . The set $\{A \rightarrow^S B\}$ is a reinstatement set for $A \rightarrow^S B$.
- **Inductive case:** Suppose for each X, Y s.t. $X \in S$, X attacks Y and S i -defends (X, Y) within k -steps, $k \geq 0$, there is a reinstatement set for $X \rightarrow^S Y$.

Suppose S i -defends (A, B) within $(k+1)$ -steps. There are two cases:

- S i -defends (A, B) within k -steps. By the inductive hypothesis there is a restatement set for $A \rightarrow^S B$.
- For each argument C attacking (A, B) there is a $D \in S$ s.t. D attacks C and S i -defends (D, C) within k -steps. By Lemma D.2 $D \rightarrow^S C$. By the inductive hypothesis, there is a reinstatement set RS_C for $D \rightarrow^S C$. From the boundedness of the EAF , it follows that $A \rightarrow^S B$ together with the union of all such RS_C form a reinstatement set for $A \rightarrow^S B$.

So there always exists a restatement set for $A \rightarrow^S B$. ■

Now we prove Lemma 6.3.

Let S be a m-conflict free set of arguments and A be an argument s.t. A is i-acceptable wrt S . We need to prove that A is m-acceptable wrt S .

Let B be an argument s.t. $B \rightarrow^S A$, i.e. B attacks A and there is no $C \in S$ s.t. C attacking (B, A) . Hence there must exist $C \in S$ attacking B and (C, B) is i-defended by S . By Lemma D.2, $C \rightarrow^S B$. By Lemma D.3 there is a reinstatement set for $C \rightarrow^S B$.

Therefore A is m-acceptable wrt S .

D.7 Proof of Lemma 6.4

1. Firstly we prove that S is m-conflict free. Assume the contrary. Hence there exist $A, B \in S$ s.t. $(A, B) \in Att$ and there is no $C \in S$ s.t. $(C, (A, B)) \in Att$. Because B is i-acceptable wrt S , there exists $D \in S$ s.t. $(D, (A, B)) \in Att$ or $(D, A) \in Att$. It follows that (D, A) is i-defended by S , contradicting the i-conflict freeness of S .
2. Let $A \in S$. Hence A is i-acceptable wrt S . From Lemma 6.3, A is m-acceptable wrt S .

D.8 Proof of Theorem 6.4

Lemma D.4 is needed to prove theorem 6.4.

Lemma D.4 *Given $S \subseteq AR$. If S is m-conflict free, then S is i-conflict free.*

Proof: Let S be a m-conflict free set. Suppose S is not i-conflict free. There exist $A, B \in S$ s.t. (A, B) is i-defended by S within n -steps. We prove by induction on n that such a case is not possible.

Basic case: $n=0$. Hence there is no argument C s.t. $(C, (A, B)) \in Att$. Contradiction with the m-conflict freeness of S .

Suppose there are no $X, Y \in S$ s.t. X attacks Y while (X, Y) is i-defended by S within n -steps.

Consider $A, B \in S$ s.t. $(A, B) \in Att$ is i-defended by S within $(n+1)$ -steps. As S is m-conflict free, there exists $C \in S$ s.t. $(C, (A, B)) \in Att$. As (A, B) is i-defended within $(n+1)$ -steps, there exists $D \in S$ s.t. $(D, C) \in Att$ and (D, C) is i-defended within n -steps. Contradiction !

■

Let R be a m-preferred extension. From Lemma D.4 above, R is i-conflict free. Let S, S' be i-admissible sets s.t. $S \cup S' \subseteq R$. Hence each argument $A \in S \cup S'$ is i-acceptable wrt $S \cup S'$. Furthermore $S \cup S'$ is i-conflict free because otherwise there exist arguments $A, B \in S \cup S'$ s.t. $(A, B) \in \Delta(S \cup S') \subseteq \Delta(R)$ (Lemma 3.1), i.e. R is not i-conflict free. Let Σ be the union of all i-admissible subsets of R (Note that since \emptyset is always i-admissible, Σ is always defined). Hence Σ is i-admissible. Because R is m-preferred, Σ is mi-admissible. Σ is hence the greatest mi-admissible set of arguments that is a subset of R .

E Proofs of Lemmas and Theorems in Section 7

E.1 Proof of Lemma 7.1

We first prove the case: if $S \cup R$ is g-complete, then $R = \Delta(S)$.

1. $\Delta(S) \subseteq R$. This property follows from Lemma B.5.
2. If $\alpha_0 \in R$, then $\alpha_0 \in \Delta(S)$.

We prove by induction on $n = \text{rank}(\text{target}(\alpha_0))$.

Basic case $n = 0$: Assume there is $\beta = (B, \alpha_0) \in \text{Att}$. It follows that $\text{rank}(B) < \text{rank}(\text{target}(\alpha_0)) = 0$. This impossibility concludes that α_0 is not attacked by any attack. So $\alpha_0 \in \Delta(S)$.

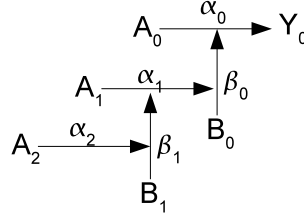
Induction hypothesis: Suppose that $\forall \alpha \in R$, if $\text{rank}(\text{target}(\alpha)) < n$ then $\alpha \in \Delta(S)$.

Consider $\alpha_0 = (A_0, Y_0) \in R$ and assume that $\text{rank}(\text{target}(\alpha_0)) = n$ and $\alpha_0 \notin \Delta(S)$. There exists $\beta_0 = (B_0, \alpha_0) \in \text{Att}$ since otherwise S i-defends α_0 within 0-steps and hence $\alpha_0 \in \Delta(S)$. As α_0 is g-acceptable wrt $S \cup R$ but not i-acceptable wrt S , by definitions of g-acceptability and i-acceptability there exists an attack $\alpha_1 = (A_1, Y_1) \in R$ for either $Y_1 = B_0$ or $Y_1 = \beta_0$, however $\alpha_1 \notin \Delta(S)$.

$Y_1 = B_0$ is not possible, since it leads to a contradiction: $\text{rank}(\text{target}(\alpha_1)) = \text{rank}(B_0) < \text{rank}(\text{target}(\alpha_0)) = n$ and hence $\text{rank}(\text{target}(\alpha_1)) < n$, concluding that $\alpha_1 \in \Delta(S)$ by the induction hypothesis.

Consider α_1 analogously to α_0 , there exists $\beta_1 = (B_1, \alpha_1) \in \text{Att}$ and $\alpha_2 = (A_2, \beta_1)$. Further, like α_0 and α_1 , $\alpha_2 \in R$ but $\alpha_2 \notin \Delta(S)$.

Reasoning this way we find an infinite sequence $\alpha_0, \beta_0, \alpha_1, \beta_1 \dots \alpha_i, \beta_i \dots$, where: $\alpha_i = (A_i, \beta_{i-1})$, $\beta_i = (B_i, \alpha_i)$, as illustrated the figure below.



Let k be a number s.t. $\alpha_0 \in Att^{k_0}$ (see definition 2.1) . Hence $\alpha_i \in Att^{k_0+2i}$. Therefore the set $\{\alpha \in Att \mid target(\alpha) = target(\alpha_0)\} \supseteq \{\alpha_k \mid k > 0\}$ is infinite. Contradiction with the strongly bounded condition.

In conclusion, $R = \Delta(S)$.

Now we prove the case: if $S \cup R$ is bccg-complete, then $R = \Pi(S)$.

By Lemma D.1: $S \cup R \subseteq F_G(S \cup R)$. By Lemma 6.2 property 2: $F_G(S \cup R) \setminus (S \cup R) \subseteq Att$. Thus we can write $F_G(S \cup R) = S \cup R \cup R_0$ where $R_0 \subseteq Att$ and $R_0 \cap R = \emptyset$.

By theorem 6.1, $F_G(S \cup R)$ is g-complete. Hence by the above proof, $R_0 \cup R = \Delta(S)$.

By Lemma 6.2, property 3: if $\alpha \in F_G(S \cup R) \setminus (S \cup R) = R_0$ then $source(\alpha) \notin S$. It follows that $\Pi(S) \subseteq R$.

From Lemma 6.2, property 1: $\alpha \in R \Rightarrow source(\alpha) \in S \Rightarrow R \subseteq \Pi(S)$.

Hence $R = \Pi(S)$.

E.2 Proof of Theorem 7.1

The theorem follows directly from Lemma 7.1 and theorems 4.1 and 6.2.

E.3 Proof of Theorem 7.2

Let EAF be a stratified MEAF with a partition $AR = AR_0 \cup AR_1 \cup AR_2 \cup \dots$

First we prove the following lemmas.

Lemma E.1 *An i -admissible set of arguments is m -conflict free.*

Proof: Let S be an i -admissible set of arguments. Assume the contrary that S is not m -conflict free. There exist $A, B \in S$ such that $(A, B) \in Att$ and $(B, A) \in Att$ or there is no $C \in S$ attacking (A, B) . If it is the latter case then from the i -acceptability of B wrt S , there is $C \in S$ s.t. C attacks A and S i -defends (C, A) . Contradicts to the i -conflict freeness of S .

Hence $(B, A) \in Att$. It follows that there is i such that $A, B \in AR_i$. Because S is i -admissible and $A \in S$, there exists $A_1 \in S$ s.t. $(A_1, B) \in Att$

is i -defended by S or A_1 attacks (B, A) . Because S is i -conflict free, the first case is not possible. Therefore A_1 attacks (B, A) . Similarly, as $B \in S$, there exist $B_1 \in S$ attacking (A, B) . From Definition 2.1, A_1 and B_1 attack each other and hence $0 \leq \text{rank}(A_1) = \text{rank}(B_1) < i$.

Analogously, there exist A_2, B_2 s.t. A_2 attacks (B_1, A_1) , B_2 attacks (A_1, B_1) , A_2, B_2 attack each other, and $0 \leq \text{rank}(A_2) = \text{rank}(B_2) < \text{rank}(A_1) < i$.

Continue and there must exist infinitely many distinct ranks between 0 and i . Contradiction!

Therefore S is m -conflict free. ■

Lemma E.2 *Let $S \subseteq AR$ be an i -conflict free and m -conflict free set of arguments and $A \in AR$ be an argument.*

Then A is i -acceptable wrt S iff A is m -acceptable wrt S .

Proof:

\Rightarrow : Suppose A is i -acceptable wrt S . As S is m -conflict free, by Lemma 6.3, A is m -acceptable wrt S .

\Leftarrow : Suppose A is m -acceptable wrt S . We prove that A is i -acceptable wrt S .

Let B be an argument attacking A . We need to prove that there is $C \in S$ such that

- C attacks B and S i -defends (C, B) or
- C attacks (B, A) . Clearly S i -defends $(C, (B, A))$ within 0-steps.

If there is some $C \in S$ attacking (B, A) then we are done. If this is not the case, we show that there is $C \in S$ attacking B and S i -defends (C, B) as follows.

Clearly $B \rightarrow^S A$. Because A is m -acceptable to S , there must exist $C \in S$ s.t. $C \rightarrow^S B$ and there is a reinstatement set RS for $C \rightarrow^S B$. We prove that S i -defends (C, B) .

From the definition of reinstatement set, $C \rightarrow^S B \in RS$ and for each $X \rightarrow^S Y \in RS$, if Y' attacks (X, Y) then there is $X' \in S$ s.t. $X' \rightarrow^S Y' \in RS$. We partition attacks in RS as follows:

- RS_0 is the set of all $X \rightarrow^S Y \in RS$ such that there is no argument attacking (X, Y) .

- RS_{k+1} is the set of all $X \rightarrow^S Y \in RS$ such that $X \rightarrow^S Y \notin RS_0 \cup \dots \cup RS_k$ and if (X, Y) is attacked by an argument Y' then there is $X' \in S$, $X' \rightarrow^S Y' \in RS_0 \cup \dots \cup RS_k$.

As EAF is hierarchical, for $k \neq k'$, RS_k and $RS_{k'}$ are disjoint. RS is finite, so the partition is finite, i.e. there is k s.t. $RS = RS_0 \cup \dots \cup RS_k$, $RS_k \neq \emptyset$.

Clearly each attack (X, Y) s.t. $X \rightarrow^S Y \in RS_0$ is i-defended by S within 0-steps, ..., each attack (X, Y) s.t. $X \rightarrow^S Y \in RS_k$ is i-defended by S within k -steps. In other words, each (X, Y) s.t. $X \rightarrow^S Y \in RS$ is i-defended by S and so is (C, B) .

■

The following lemma holds.

Lemma E.3 *Let $S \subseteq AR$ be a set of arguments. Then S is m-admissible iff S is i-admissible.*

Proof:

\Rightarrow : Suppose S is i-admissible.

By Lemma E.1, S is m-conflict free.

Let $A \in S$. A is i-acceptable wrt S . By Lemma E.2, A is m-acceptable wrt S . It means that $S \subseteq F_M(S)$, i.e. S is m-admissible.

\Leftarrow : Suppose S is m-admissible.

As S is m-conflict free, by Lemma D.4, S is i-conflict free.

Let $A \in S$. A is m-acceptable wrt S . By Lemma E.2, A is i-acceptable wrt S . It means that $S \subseteq F_I(S)$, i.e. S is i-admissible.

■

Now we prove Theorem 7.2.

Let $S \subseteq AR$ be a set of arguments.

S is an i-complete extension

iff S is i-admissible and $S = F_I(S)$

iff S is i-admissible and $A \in S \Leftrightarrow A$ is i-acceptable wrt S

iff S is m-admissible (by lemma E.3) and $A \in S \Leftrightarrow A$ is m-acceptable wrt S (by Lemma E.2)

iff S is m -admissible and $S = F_M(S)$
iff S is a m -complete extension.