A Sound and Complete Dialectical Proof Procedure for Sceptical Preferred Argumentation

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Abstract. We present a dialectical proof procedure for computing skeptical preferred semantics in argumentation frameworks. The proof procedure is based on the dispute derivation introduced for assumption-based framework. We prove the soundness of the procedure for any argumentation frameworks and the completeness for a general class of finitary argumentation frameworks containing the class of finite argumentation frameworks as a subclass.

1 Introduction

Argumentation is a form of reasoning, that could be viewed as a debate, in which the participants present their arguments to establish, defend, or attack certain propositions. An argument could be said to represent a consensus if it is accepted by all participants. For example, in legal domain, different members of a jury could have different views of the presented evidence (different preferred extensions) but a guilty verdict is the result of a consensus among members. This form of reasoning to find a consensus is characterized by the skeptical semantics in argumentation. Skeptical semantics is also useful in AI systems for negotiation and decision making [14–17, 21].

Several procedures for computation of skeptical preferred semantics have been proposed, e.g the TPI procedure [19] for coherent argumentation framework [4, 10], and a dialectic procedure for finding ideal skeptical semantics, an approximation of the skeptical preferred semantics [9]. In [4], an algorithm for computing sceptical preferred semantics is proposed. Given an argument a, the algorithm proceeds in two separate steps: It first checks that a is not attacked by any admissible set. In the second step, it looks for an admissible set that can not be extended into a bigger one containing a. Failure to find such a set implies that a is included in each preferred extension. In other words, the algorithm represents an indirect way of proving that a is skeptically preferred based on the idea that failure to show that a is not skeptical preferred implies the contrary. Though the idea is intuitively clear, no formal proof for the soundness of the algorithm is given.

In contrast, in this paper we present a direct dialectical proof procedure for general skeptical preferred semantics. We prove the soundness of the procedure for any argumentation frameworks and its completeness for a general class of finitary argumentation frameworks containing the class of finite argumentation frameworks as a subclass.

The structure of the paper is as follows. In section 2 we recall and introduce notions of proof tree, proof derivation and proof procedure for credulous preferred semantics. We introduce finitary argumentation frameworks and prove soundness and completeness of credulous proof procedure for them. In section 3 we present proof theories and algorithm for general skeptical preferred semantics.

2 Credulous Acceptance

Following [7], we define an argumentation framework as a pair $AF = (\mathcal{A}, att)$, where A is a set of arguments, and att is a binary relation on $\mathcal{A}$ ($att \subseteq \mathcal{A} \times \mathcal{A}$). Given two arguments A and B, (A,B)$\in$ att means A attacks B. A set S of arguments attacks an argument A if there is an argument B in S such that B attacks A. The definitions of conflict-free set, admissible set and preferred extension are recalled from [7] as follows:

Let S be a set of arguments

1. S is conflict-free iff there exist no arguments A, B in S such that A attacks B
2. Argument A is acceptable with respect to S iff for each argument B if B attacks A then S attacks B
3. S is admissible iff S is conflict-free and each argument in S is acceptable with respect to S
4. S is a preferred extension of AF iff S is a maximal admissible set of AF
5. Argument A is credulously accepted iff A is contained in at least one preferred extension of AF
6. Argument A is skeptically accepted iff A is contained in every preferred extensions of AF

To prove the credulous acceptance of an argument, a proof tree is constructed. A proof tree can be viewed as a specification of a debate between a proponent and an opponent, where an initial argument is put forward by the proponent, and then the opponent and proponent alternatively present their arguments to attack the arguments of the other. The proponent wins the dispute if he can attack every attacking argument of the opponent. We recall the definition of proof tree from [8, 9]:

Definition 1. *A proof tree for an argument A with respect to an argument framework AF is defined as following:*

1. *Nodes are labeled by arguments and the root is labeled by A. The argument labeling a child node attacks the argument labeling its parent.*

2. *There are two types of nodes: proponent nodes and opponent nodes*
3. *Each opponent node has exactly one child that is a proponent node*
4. *For each proponent node N labeled by an argument B, N has as many children nodes as the number of arguments attacking B, and for every argument C attacking B, there is a child node of N, which is an opponent node labeled by C.*

Definition 2. *A proof tree is said to be admissible if there is no argument that labels both a proponent node and an opponent node.*

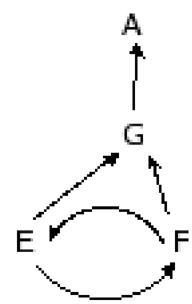

Fig. 1.

Fig. 2.

Example 1. The argumentation framework AF=($\mathcal{A}$,att) is depicted on Fig. 1, where: $\mathcal{A}$={A,G,E,F} and att = {(G, A),(E, G),(F, G),(E,F),(F, E)}.

A proof tree for argument A is depicted in Fig. 2 where:

– argument A labels the root
– arguments A, E label proponent nodes
– arguments G, F label opponent nodes
– this tree is admissible, since there is no argument labeling both opponent and proponent nodes.

The following lemma is similar to a theorem from [8] and proved in the extended version of [9].

Lemma 1.

1. *Let T be an admissible proof tree for A and S the set of all arguments labeling proponent nodes of T. Then S is admissible.*
2. *Let S be an admissible set of arguments and A in S. Then there exists an admissible proof tree T for A such that the set of arguments labeling the proponent nodes in T is a subset of S*

A proof tree is often infinite as example 1 shows. A proof derivation is a finite top-down construction of an (possibly infinite) admissible proof tree by a sequence of tuples $< P, O, SP, SO >$, where P is a set of arguments, put forward by the proponent but which have not been attacked yet, and O is a set of arguments put forward by the opponent to attack the proponent's arguments, against which the proponent doesn't have counter-attack until now. SP is the set of arguments presented by proponent, and SO is the set of arguments put forward by the opponent, and already counter-attacked by the proponent. Our proof derivation is defined in a spirit like the dispute derivation in [8] and the dialectical games in [4]. In each step of a proof derivation building process only one argument is selected. Let B be a selected argument and let O_B be the set of all arguments attacking B. In the first case if B labeling a proponent node, then O_B consists of all arguments labeling opponent child nodes of the node labeled by B. In the second case if B labeling an opponent node, then O_B is a set of arguments, from which one argument is chose to label a proponent child node of the node labeled by B. Hence there exists no proof derivation if there is one argument in O_B labeling a proponent node in the first case, or there is one argument in O_B labeling an opponent node or $O_B = \emptyset$ in the second case, because our proof tree is not admisible.

Definition 3. *A proof derivation $\mathcal{D}$ for an argument A is a sequence $< P_0, O_0, SP_0, SO_0 > ... < P_n, O_n, SP_n, SO_n >$ where:*

1. *P_i, O_i, SP_i, and SO_i are argument sets*
2. $P_0 = SP_0 = \{A\},\ SO_0 = O_0 = \emptyset,\ P_n = O_n = \emptyset$
3. *Let B be the argument selected at step i, and let O_B be the set consisting of all arguments attacking B.*
   (a) *If $B \in P_i$ and $O_B \cap SP_i = \emptyset$ then*
   $P_{i+1} = P_i \setminus \{B\}$
   $O_{i+1} = O_i \cup (O_B \setminus SO_i)$
   $SP_{i+1} = SP_i$
   $SO_{i+1} = SO_i$
   (b) *If $B \in O_i$ then select an argument $C \in O_B$ such that $C \notin (SO_i \cup O_i)$*
   $P_{i+1} = Pi \cup \{C\}$ *if* $C \notin SP_i$, *otherwise* $P_{i+1} = P_i$
   $O_{i+1} = O_i \setminus \beta$ *where* $\beta = \{B' \mid C \text{ attacks } B'\}$
   $SP_{i+1} = SP_i \cup \{C\}$
   $SO_{i+1} = SO_i \cup (\beta \cap O_i)$ *(Note that* $B \in \beta \cap O_i$*)*

Example 2. Let argumentation framework AF=($\mathcal{A}$,att), where $\mathcal{A} = \{A\}$ and att=$\emptyset$ then a sequence $< \{A\}, \emptyset, \{A\}, \emptyset >< \emptyset, \emptyset, \{A\}, \emptyset >$ is the proof derivation for A.

Example 3. (*Continue example 1*) A proof derivation for A is presented in following table, where the notation X̲ means that X is selected in step 3 of definition 3.

i	$\mathbf{P_i}$	$\mathbf{O_i}$	$\mathbf{SP_i}$	$\mathbf{SO_i}$	comment
0	A̲	$\emptyset$	A	$\emptyset$	O_A={G} according to step3.a
1	$\emptyset$	G̲	A	$\emptyset$	O_G={E,F}, E is selected form O_G, β={G,F} according to step3.b
2	E̲	$\emptyset$	A, E	G	O_E={F} according to step3.a
3	$\emptyset$	F̲	A, E	G	O_F={E}, E is selected and E$\in$ SP_3, β={G,F} according to step3.b
4	$\emptyset$	$\emptyset$	A, E	G, F	

Table 1. The construction of a proof derivation for A

Theorem 1.

1. *Suppose* $< P_0, O_0, SP_0, SO_0 > ... < P_n, O_n, SP_n, SO_n >$ *is a proof derivation for A. Then* SP_n *is admissible and* $A \in SP_n$.

2. *Let AF be a finite argumentation framework, and let A be an argument of AF. If A belongs to an admissible set then there is a proof derivation for A.*

Consider the infinite argumentation framework in Fig. 3. It is not difficult to see that there is an unique preferred extension consisting of arguments $A_0, A_2, ..., A_{2n}$, ... It is obvious that for each argument A_{2n} there is a proof derivation for A_{2n}. The reason for the existence of a proof derivation for A_{2n} is that the argumentation framework consisting of the arguments from which there is a directed path to A_{2n} is finite. In the following, we introduce the class of finitary argumentation frameworks generalizing this property.

Fig. 3.

Let AF=($\mathcal{A}$, att) and $A \in \mathcal{A}$.[1] The environment of A denoted by ENV_A is the set of all arguments B in $\mathcal{A}$ such that there is a directed path from B to A in the graph of AF (i.e.there is a sequence $B_1, B_2..., B_n$ such that B_i attacks B_{i+1} and B=B_1 and A=B_n). Let $AF_A = (ENV_A, att_A)$, where att_A is the restriction of att to ENV_A.

[1] For purpose of reference, we often identify AF with the graph representing it.

Definition 4. *An argumentation framework is said to be finitary if for each argument A, AF_A is finite.*

Lemma 2.

1. *Let S be an admissible set of arguments in AF. Then $S \cap ENV_A$ is also admissible in both AF and AF_A.*
2. *Let $S \subseteq ENV_A$ be an admissible set in AF_A. Then S is also admissible in AF.*

From the lemma 2, it is obvious that

Corollary 1. *A is credulously accepted in AF iff A is credulously accepted in AF_A.*

The soundness and completeness of proof derivation for finitary argumentation frameworks follows immediately from the above corollary and theorem 1.

Theorem 2. *Let AF be a finitary argumentation framework, and A be an argument of AF. A belongs to an admissible set iff there is a proof derivation for A.*

3 Skeptical Acceptance

An argumentation framework *AF* is said to be coherent if each preferred extension of *AF* is stable. In other words, coherence implies the coincidence between stable and preferred semantics. TPI procedures are based on the following proposition [4, 11, 19] to check whether a given argument A is skeptically accepted in coherent argumentation frameworks: An argument A is skeptically accepted in a coherent argumentation frameworks if A is credulously accepted and there exists no admissible set attacking A.

The following example shows that TPI procedures can not be used for answering whether a given argument belongs to all preferred extensions in general cases.

Example 4. The argumentation framework $AF=(\mathcal{A},att)$ is depicted in Fig. 4, where $\mathcal{A}$ = {A,B,G,E,F} and *att* = {(G,A),(A,B),(B,G),(E,G),(F,E),(E,F)}

It is clear that {A,E} and {F} are the only preferred extensions, and argument A is not skeptically accepted, although A is credulously accepted and there exist no admissible set attacking A.

In this chapter we introduce a proof procedure for skeptical preferred semantics in general cases, which is based on the following simple lemma.

Lemma 3. *Let S be an admissible set of arguments and E be a preferred set of arguments, and S is not a subset of E. Then E attacks S (and S also attacks E).*

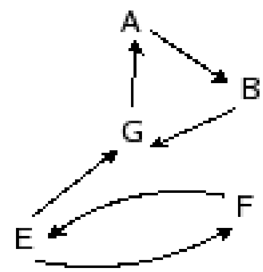

Fig. 4.

Definition 5. *Let A be an argument, and let $\mathcal{B}$ be a set of admissible sets such that each element of $\mathcal{B}$ contains A.*

1. *If for each preferred extension E such that A$\in$ E, there exists an admissible set $S \in \mathcal{B}$ such that $S \subseteq E$ then $\mathcal{B}$ is called a base of A.*
2. *A base $\mathcal{B}$ of A is said to be complete if for each preferred extension E, there is a set S$\in$ B such that $S \subseteq E$*

Lemma 4. *(Skeptical Lemma) An argument A is sceptically accepted iff there exist a complete base $\mathcal{B}$ of A.*

The skeptical lemma suggests that a proof procedure for showing that A is skeptically accepted, could proceed in two steps:

1. Generate a base $\mathcal{B}$ of A
2. Verify that $\mathcal{B}$ is a complete base of A

3.1 Generating a Base of A

We define a $\mathcal{BG}$[2]-derivation for an argument A by constructing all possible proof derivations for A.

Definition 6. *$\mathcal{BG}$-derivation for A is a sequence $T_0, T_1, ..., T_n$, where:*

1. *T_i is a set of tuples of the form $< P, O, SP, SO >$*
2. *$T_0 = \{< \{A\}, \emptyset, \{A\}, \emptyset >\}$*
3. *Each tuple t of T_n has the form $< \emptyset, \emptyset, SP, SO >$*
4. *At each step T_i one tuple $t_i =< P_i, O_i, SP_i, SO_i >$ is selected from T_i and one argument B is selected from P_i or O_i.*
   (a) *If B is selected from P_i, then: $T_{i+1} = (T_i \setminus \{t_i\}) \cup \{t'\}$, where t' is computed from t_i as in definition 3 step 3.a.*

[2] $\mathcal{BG}$ stands for "Base Generation"

(b) If B is selected from O_i and let O_B be the set consisting of all arguments attacking B, then: $T_{i+1} = (T_i \setminus \{t_i\}) \cup \{t' \mid t'$ is computed from t_i as in definition 3 step 3.b for some argument $C \in O_B$ such that $C \notin (SO_i \cup O_i)\}$

It is not difficult to see the following:

Theorem 3.

1. *Let $T_0, T_1, ..., T_n$ be a BG-derivation for A. Let $\mathcal{B} = \{SP \mid < \emptyset, \emptyset, SP, SO > \in T_n\}$. Then $\mathcal{B}$ is a base of A.*
2. *Let AF be finitary. Then there exists a BG-derivation for A.*

3.2 Verifying the Completion of a Base

Before giving the procedure for verifying the completeness of a base, we need a few technical results.

Lemma 5. *Let $\mathcal{B}$ be a base of argument A. $\mathcal{B}$ is a complete base of A iff there exist no preferred extension E attacking every element of $\mathcal{B}$.*

A proof derivation for a given argument A is constructed to find an admissible set of arguments defending A. However in some cases we want to answer the question "can the proponent admissibly attack arguments proposed by the opponent". A notion of a proof derivation $\mathcal{D}$ against S is introduced for this purpose.

Definition 7. *A proof derivation $\mathcal{D}$ against a set S of arguments is defined as a sequence $< P_0, O_0, SP_0, SO_0 > ... < P_n, O_n, SP_n, SO_n >$ where:*

1. *$P_i, O_i, SP_i,$ and SO_i are argument sets*
2. *$P_0 = SP_0 = \emptyset, O_0 = S, SO_0 = \emptyset, P_n = O_n = \emptyset$*
3. *$< P_{i+1}, O_{i+1}, SP_{i+1}, SO_{i+1} >$ is constructed from $< P_i, O_i, SP_i, SO_i >$ as in definition 3 step 3.*

Lemma 6. *For finitary argumentation frameworks, there exists a proof derivation $\mathcal{D}$ against a set S iff there exist an admissible set S' attacking every element in S.*

Let A be an argument and $\mathcal{B} = \{S_1, ..., S_n\}$ where S_i is an admissible set containing A, and let $\mathcal{CB} = \{S \mid \exists e \in S_1 \times S_2 \times ... \times S_n$ and S is the set of arguments appearing in e$\}$, and let $\mathcal{XB} = \{S \mid S \in \mathcal{CB}$ and S is minimal in $\mathcal{CB}$ wrt set inclusion$\}$

Lemma 7. *For finitary argumentation frameworks, let $\mathcal{B}$ be a base of A. $\mathcal{B}$ is a complete base of A iff for each S$\in \mathcal{XB}$ there exist no proof derivation $\mathcal{D}$ against S.*

Based on lemma 7 we define now a $\mathcal{CB}$[3]-verification for a base $\mathcal{B}$ of an argument A to verify the completeness condition of $\mathcal{B}$.

Definition 8. *Let $\mathcal{B}$ be a finite set. A* <u>$\mathcal{CB}$*-verification*</u> *for $\mathcal{B}$ is a sequence $J_0, J_1...J_n$ where*

1. *J_i is a set of tuples of the form $< P, O, SP, SO >$*
2. *$J_0 = \{< \emptyset, O, \emptyset, \emptyset > | \; O \in \mathcal{XB}\}$*
3. *$J_n = \emptyset$*
4. *J_{k+1} is obtained from J_k like T_{k+1} is obtained from T_k in definition 6.*

Theorem 4. *Let AF be a finitary framework and $\mathcal{B}$ be a finite base of argument A. There exists a $\mathcal{CB}$-verification for $\mathcal{B}$ iff $\mathcal{B}$ is a complete base of A.*

3.3 Proof Procedure for Skeptical Acceptance

We define a $\mathcal{SA}$[4]-derivation for A as a combination of a $\mathcal{BG}$-derivation for A and a $\mathcal{CB}$-verification for the base created by the $\mathcal{BG}$-derivation.

Definition 9. *Let A be an argument. An $\mathcal{SA}$-derivation for A is a sequence $T_0, T_1, ..., T_n, J_0, J_1...J_m$ where:*

1. *The sequence $T_0, T_1, ..., T_n$ is a $\mathcal{BG}$-derivation for A*
2. *The sequence $J_0, J_1...J_m$ is a $\mathcal{CB}$-verfication for $\mathcal{B}$, where $\mathcal{B} = \{SP \mid < \emptyset, \emptyset, SP, SO > \in T_n\}$*

The following theorem follows directly from theorems 3, 4

Theorem 5. *Let AF be a finitary argumentation framework and A be an argument in AF. A is sceptically accepted iff there exists a SA-derivation for A.*

Example 5. (*Continue example 1*) Our proof procedure shows that A is skeptically accepted (see table 2). The notion fails means 'fails to build a proof derivation'.

Example 6. (*Continue example 4*) Our proof procedure shows that A is not skeptically acceptedm(see table 3), something that can not be done using TPI-procedures.

From table 3 we see that there exist no $\mathcal{SA}$-derivation for A. Hence A is not sceptically accepted.

[3] $\mathcal{CB}$ stands for Complete Base
[4] $\mathcal{SA}$ stands for Skeptical Acceptance

$\mathcal{BG}$-derivation for A					
	P	**O**	**SP**	**SO**	**comment**
T_0	$\underline{A}$	$\emptyset$	A	$\emptyset$	step4.a, O_A={G}
T_1	$\emptyset$	$\underline{G}$	A	$\emptyset$	step4.b, O_G={E,F}
T_2	$\underline{E}$	$\emptyset$	A, E	G	step4.a, O_E={F}
	F	$\emptyset$	F, A	G	
T_3	$\emptyset$	$\underline{F}$	A, E	G	step4.b, $O_F\cap$ SP={E}
	F	$\emptyset$	F, A	G	
T_4	$\emptyset$	$\emptyset$	A, E	G	
	$\underline{F}$	$\emptyset$	A, F	G	step4.a O_F={E}
T_5	$\emptyset$	$\emptyset$	E,A	G	
	$\emptyset$	$\underline{E}$	F,A	G	step4.b $O_E\cap$SP={F}
T_6	$\emptyset$	$\emptyset$	E,A	G	$\mathcal{B}$={{E,A}, {F,A}} and
	$\emptyset$	$\emptyset$	F,A	G	$\mathcal{XB}$ ={{A},{E,F}}

$\mathcal{CB}$-verification for $\mathcal{B}$					
	P	**O**	**SP**	**SO**	**comment**
J_0	$\emptyset$	$\underline{A}$	$\emptyset$	$\emptyset$	step4.b, O_A={G}
	$\emptyset$	E, F	$\emptyset$	$\emptyset$	
J_1	$\underline{G}$	$\emptyset$	G	A	step4.a, O_G={E,F}
	$\emptyset$	E,F	$\emptyset$	$\emptyset$	
J_2	$\emptyset$	$\underline{E}$,F	G	A	step4.b $O_E\cap$O={F}, **fails**
	$\emptyset$	E,F	$\emptyset$	$\emptyset$	
J_3	$\emptyset$	$\underline{E}$,F	$\emptyset$	$\emptyset$	step4.b $O_E\cap$O={E}, **fails**
J_4	**Empty**				

Table 2. Construction of a BG for A and CB-verification for $\mathcal{B}$

	P	**O**	**SP**	**SO**	**comment**
T_0	$\underline{A}$	$\emptyset$	A	$\emptyset$	step3.a, O_A={G}
T_1	$\emptyset$	$\underline{G}$	A	$\emptyset$	step3.b, O_G={E,B}
T_2	$\underline{E}$	$\emptyset$	A, E	G	step3.a, O_E={F}
	B	$\emptyset$	A, B	G	
T_3	$\emptyset$	$\underline{F}$	A, E	G	step3.b, O_F={E}, {E}*in* SP
	B	$\emptyset$	A, B	G	
T_4	$\emptyset$	$\emptyset$	A, E	G, F	
	$\underline{B}$	$\emptyset$	A, B	G	step3.a $O_B\cap$ SP={A} **fails**
T_5	$\emptyset$	$\emptyset$	A, E	G, F	$\mathcal{B}$={{A, E}} and $\mathcal{XB}$={{A}, {E}}

	P	**O**	**SP**	**SO**	**comment**
J_0	$\emptyset$	$\underline{E}$	$\emptyset$	$\emptyset$	step3.b, O_E={F}
	$\emptyset$	A	$\emptyset$	$\emptyset$	
J_1	$\underline{F}$	$\emptyset$	F	E	step3.a, O_F=SO={E}
	$\emptyset$	A	$\emptyset$	$\emptyset$	
J_2	$\emptyset$	$\emptyset$	F	E	
	$\emptyset$	$\underline{A}$	$\emptyset$	$\emptyset$	step3.b O_A={G}
J_3	$\emptyset$	$\emptyset$	F	E	
	$\underline{G}$	$\emptyset$	G	A	step3.a O_G={E, B}
J_4	$\emptyset$	$\emptyset$	F	E	
	$\emptyset$	$\underline{B}$, E	G	A	step3.b $O_B\cap$SO={A} **fails**
J_5	$\emptyset$	$\emptyset$	F	E	**not empty**

Table 3. Construction of the SA-derivation for A

4 Conclusion and Discussions

It is a well-known result from [11] that skeptical acceptance is $\prod_2^{(p)}$-complete. Therefore in worst cases, computing a $\mathcal{SA}$ derivation is not polynomial.

Consider the argumentation framework in Fig. 5. Using the $\mathcal{BG}$-derivation, we would be able to generate a base $\mathcal{B}$ = {{A,E,C},{A,E,D},{A,F,C},{A,F,D}}. Looking at the subgraph consisting of only E,F, we could realize that if there is any attack against E or F, it should come from within this subgraph. Similarly for C,D. Hence, it would be enough if in the $\mathcal{CB}$-verification, we consider only derivations againsts {A},{E,F},{C,D}. Structuring argumentation frameworks into strongly connected component like in [1] would facilitate optimizing the $\mathcal{SA}$-derivations in this direction.

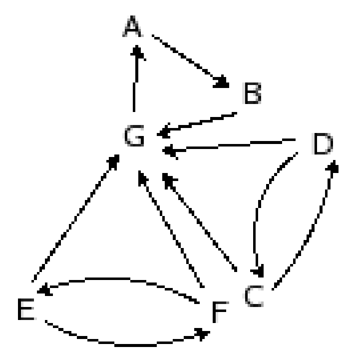

Fig. 5.

5 Acknowledgements

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A Appendix

A.1 Proof of lemma 2

1. Let R=$S \cap ENV_A$. It is obvious that R is conflict-free. Let B be an argument attacking R. It is obvious that B $\in ENV_A$. Hence there is C $\in S$ such that C attacks B. Hence C $\in ENV_A$. Hence C $\in R$. Hence R is admissible both wrt AF and AF_A.
2. It is clear that S is conflict-free in AF. Let B be an argument attacks S in AF. Hence B $\in ENV_A$. Hence S attacks B in AF_A. Hence S attacks B in AF.

A.2 Proof of lemma 3

It is clear that if E attacks S then S also attacks E and vice versa. Assume that S, E do not attack each other. Hence C=S ∪ E is conflict-free. For each argument A in C if there is an argument B attacking A then B is attacked by S or by E since A is in S or E. So B is attacked by C. Hence each argument in C is acceptable wrt C. Then C is admissible and contains E and there exists an argument G in C which is not in E because S is not subset of E. Contradiction since E is preferred. Hence S attacks E and E also attacks S.

A.3 Proof of lemma 4

1. Only if part
   Let $\mathcal{B}$ be the set of all preferred extensions, then $\mathcal{B}$ is a complete base of A.
2. If part
   Let $\mathcal{B}$ be a complete base of A, then for each preferred extension E there exists a set $S \in \mathcal{B}$ such that $S \subseteq E$. Since A $\in S$ for each $S \in \mathcal{B}$ then A is contained in each preferred extension E. Hence A is sceptically accepted.

A.4 Proof of lemma 5

1. Only if part
   Let $\mathcal{B}$ be a complete base of A, and E be an arbitrary preferred extension. Then there is an admissible set S of $\mathcal{B}$ such that $S \subseteq E$. Hence E does not attack S. Hence E does not attack every element of $\mathcal{B}$.
2. If part
   Assume $\mathcal{B}$ is not a complete base of A. Then there exists a preferred extension E such that $E \not\supseteq S$ for every element S of $\mathcal{B}$. Hence E attacks every element S of $\mathcal{B}$ (lemma 3). Contradiction.

A.5 Proof of lemma 6

Let AF=($\mathcal{A}$,att) be the argumentation framework we are working in. Let AF'=($\mathcal{A}'$,att') be another argumentation framework such that $\mathcal{A}' = \mathcal{A} \cup$ {T}, where T is a new argument not in $\mathcal{A}$, $att' = att \cup$ {(C, T) | C $\in S$}. Then each proof derivation $\mathcal{D}$ against S can be transferred into a proof derivation $\mathcal{D}'$ for T by adding the tuple $< \{T\}, \emptyset, \{T\}, \emptyset >$ to the beginning of $\mathcal{D}$ and add T to the SP component in each tuple in $\mathcal{D}$

1. Only if part
   Since there is a proof derivation $\mathcal{D}$ against a set S in AF, then there is a proof derivation $\mathcal{D}'$ for T in AF'. Hence there exists an admissible set R in AF' containing T. Since T is attacked by every element of S, then each element of S is attacked by R. Let $S' = R \setminus$ {T}. Hence each element of S is attacked by S'. S' is conflict-free, because R is an admissible set. Since $S' \subseteq \mathcal{A}$, every argument attacking S' belongs to $\mathcal{A}$. For each argument B

attacking *S'*, there is an argument B' $\in$ *S'* such that (B',B)$\in$ *att*, because *att'*=*att* $\cup$ {(C, T) | C $\in$ *S*). Hence *S'* is an admissible set wrt *AF*. So there exists an admissible set attacking every element in *S*.
2. If part
Let R=*S'* $\cup$ {T}. T is not in $\mathcal{A}$, then T is not in *S'*. Furthermore *S'* is admissible, and set *S'* defends T, then R is admissible. Hence there is a proof derivation $\mathcal{D}'$ for T. Hence there is a proof derivation $\mathcal{D}$ against a set S by dropping the first tuple from $\mathcal{D}'$.

A.6 Proof of lemma 7

1. Only if part
Let E be a preferred extension. Since $\mathcal{B}$ is a complete base for A, then there is a set $S_i \in \mathcal{B}$ such that $S_i \subseteq E$. That means for each $S \in \mathcal{XB}$ there is an argument C $\in$ ($S \cap S_i$) such that E doesn't attack C. Hence for each $S \in \mathcal{XB}$ there exist no preferred extension attacking every element in S. Then for each $S \in \mathcal{XB}$ there exists no admissible set attacking every element in S. Hence there exists no proof derivation $\mathcal{D}$ against S (lemma 6).
2. If part
Assume the contradiction, that means $\mathcal{B}$ is not complete base of A. Hence there exists a preferred extension E attacking every element S_i of $\mathcal{B}$ (lemma 5). Hence for each S_i there is an argument C_i in S_i such that E attacks C_i. Hence E attacks every element in S=$\{C_1, C_2 ... C_n\} \in \mathcal{XB}$. Hence there exists proof derivation $\mathcal{D}$ against S (lemma 6). Contradiction.

A.7 Proof of theorem 4

Let $\mathcal{XB} = \{O_1, ..., O_n\}$. Let AF=($\mathcal{A}$,*att*). Let AF'=($\mathcal{A}'$,*att'*) and $\mathcal{A}' = \mathcal{A} \cup R$ where R={A',Q,$G_1, ..., G_n$} and $\mathcal{A} \cap R = \emptyset$, *att'*=*att* $\cup\{(C_1, G_1) \mid C_1 \in O_1\} \cup ... \cup\{(C_n, G_n) \mid C_n \in O_n\} \cup \{(G_1, Q), ..., (G_n, Q), (Q, A')\}$ (figure 6). A $\mathcal{CB}$-verification $\mathcal{D}$ for $\mathcal{B}$ can be transferred to a proof derivation $\mathcal{D}'$ for A' by adding a sequence T_0, T_1, T_2 to the beginning of $\mathcal{D}$, and

$$T_0 =< \{A'\}, \emptyset, \{A'\}, \emptyset >$$

$$T_1 =< \emptyset, \{Q\}, \{A'\}, \emptyset >$$

$$T_2 = \{< \{G_i\}, \emptyset, \{G_i, A'\}, \{Q\} >\mid i \in [1, n]\}$$

$$T_3 = \{< \emptyset, O_i, \{G_i, A'\}, \{Q\} >\mid i \in [1, n]\}.$$

It is not difficult to see that T_3 corresponds to J_0 of $\mathcal{D}$ in the sense that the first two components of the tuples in T_3 coincide with the first two components of the tuples in $\mathcal{D}$. $\mathcal{D}$ could be easily modified to have T_3 as its first element since G_i, A', Q do not have any effects on the status of the elements in $\mathcal{A}$. Abusing the notation, we still identify T_3 and J_0.

1. Only if part

   There is a $\mathcal{CB}$-verification for $\mathcal{B}$ wrt AF. Hence there exists no proof derivation for A' in AF'. Hence there exists no admissible set containing one of $G_1, ..., G_n$. That means ∀i there is no admissible set containing G_i. Therefore ∀i there is no admissible set attacks each argument in O_i. Hence there is no admissible set attacking each element in $\mathcal{B}$. Hence $\mathcal{B}$ is complete.
2. If part

   $\mathcal{B}$ is complete then there is no admissible set attacking every $S_i \in \mathcal{B}$ wrt AF. So there is no admissible set attacking every element of O_i for each $O_i \in \mathcal{XB}$. We prove that there is no proof derivation for A' wrt AF'. Assume contradiction, that means there is an admissible set S containing A'. Let S'=$S \setminus$ {A'}. Since S is admissible and A' doesn't defend any argument, then S' is admissible. G_i defends A', then at least one $G_i \in S'$. Set O_i attacks G_i then every element of O_i is attacked by S' in AF'. Let $S''=S' \setminus \{G_i\}$. Since S' is admissible and G_i does not attack any argument in S', then S'' is admissible, and every element of O_i is attacked by S'' in AF'. Since $R \cap S''=\emptyset$ and S'' is not attacked by R and S'' is admissible, then every element of O_i is attacked by S'' in AF. Contradiction. Hence there exists no proof derivation for A'. Hence there is a $\mathcal{CB}$-verification for $\mathcal{B}$.

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