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Multi-Reference Shape Priors for Active Contours

Alban Foulonneau · Pierre Charbonnier · Fabrice Heitz

Presented by:
Mr. Parkpoom Kerdtanamongkol ID: 107610
Machine Vision for Robotics and HCI

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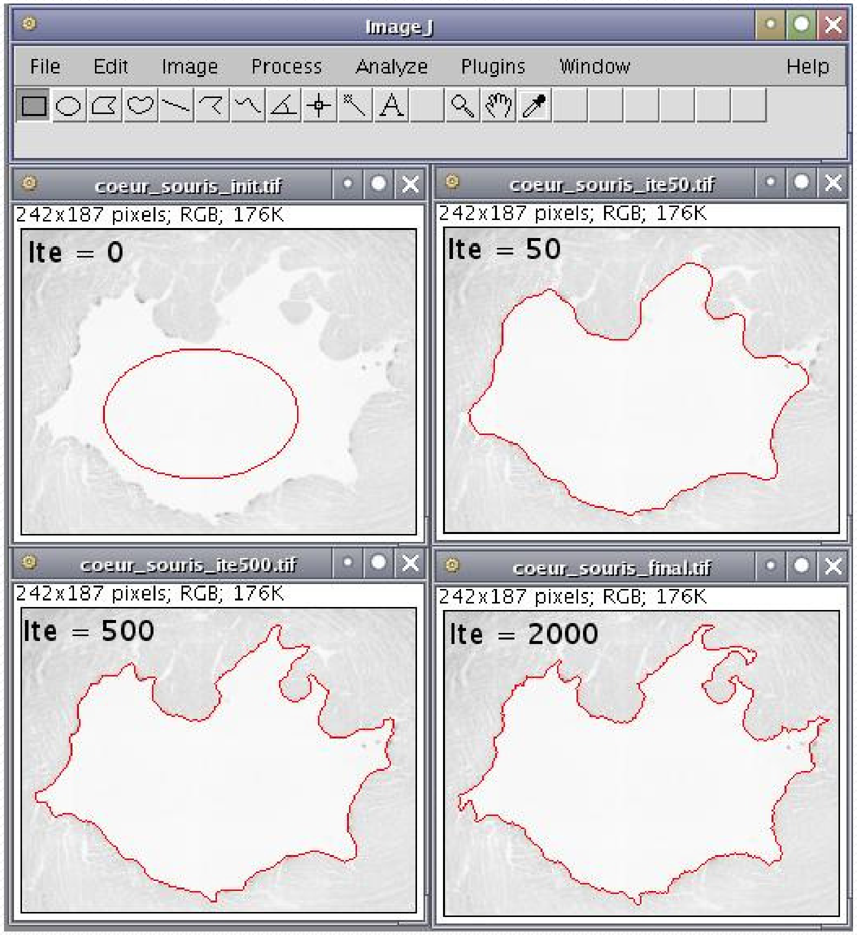
Outline

- Background
- Abstract
- Introduction
- Shape Descriptor and Geometrical Invariance
- Multi-Reference Shape Priors
- Application to Image Segmentation
- Conclusion

Background[1]

<u>Active contour:</u>

- Active contours are dynamic contour. By evolution, it moves towards the object boundaries or image features which we want to extract.

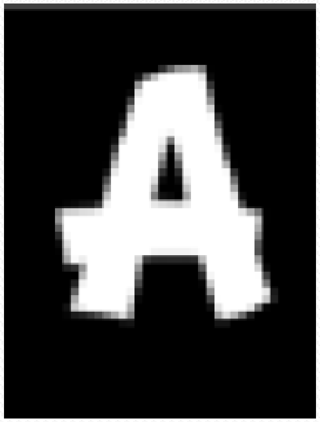


http://imagejdocu.tudor.lu/doku.php?id=plugin:segmentation:active_contour:start

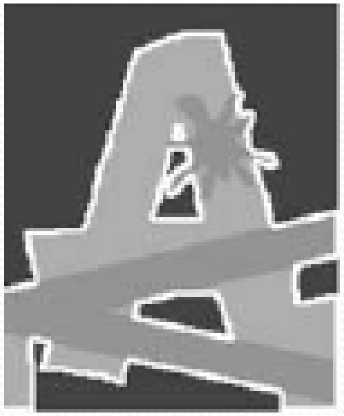
Background[2]

Reference Shape Prior:

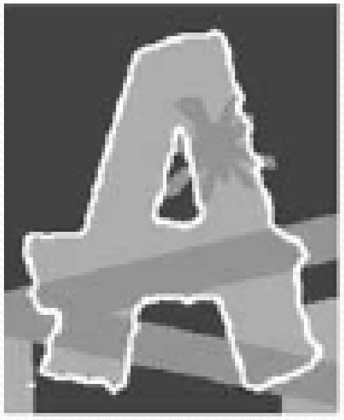
- Active contour wraps the object boundary which may contain noises.
- With shape prior, the contour can fit the object and match the reference robustly although the object contains noises.



Shape Prior



Contour without shape prior



Contour with shape prior

Background[3]

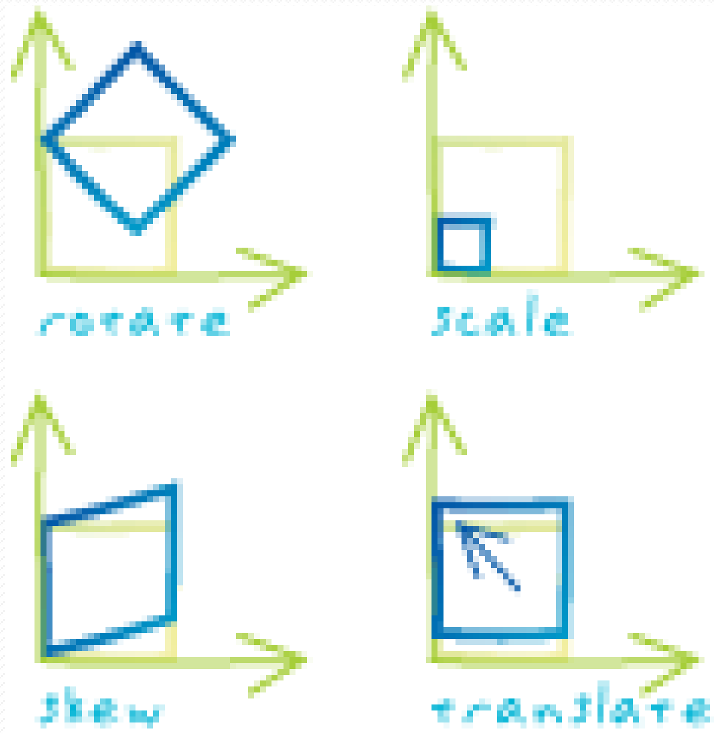
<u>Shape descriptor:</u>

- Shape descriptor is used to represent a contour or a shape. It is defined as a D-dimentional vector.
  - Non-parametric representation
  - Parametric representation by moments
    - Regular or geometric moment ($M_{p,q}$)
    - Legendre moment ($\lambda_{p,q}$)
    - Normalized central moment ($\eta_{u,v}$)

Background[4]

Affine transformation:

- Affine transformation is a geometric transformation that scales, rotates, skews, and/or translates images.
- In an affine transformation, parallel lines remain parallel, the midpoint of a line segment remains a midpoint, and all points on a straight line remain on a straight line.



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Abstract

- In this paper , we present a new way of constraining the evolution of an active contour with respect to a set of fixed reference shapes.
- Shape descriptor is based on Legendre Moments computed from their characteristic function.
- By properties of moments, it is possible to include intrinsic affine invariance in the descriptor which solve shape alignment issue.
- Our shape prior is based on a distance, in terms of descriptors, between the evolving curve and the reference shapes.

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Introduction[1]

- We present a novel approach for constraining the geometry of an evolving active contour toward a set of reference shapes.

Introduction[2]

- Relationship to prior work
  - The first issues: variability
    - When using reference shape, it must be dealt with a question about variability of reference shapes which is the variation of the shape away from reference template.
    - It is handled by using statistical models. Many models are based on standard Principal Component Analysis (PCA) (Cootes et al. 1995; Székely et al. 1996; Leventon et al. 2000; Tsai et al. 2003; Bresson et al. 2006) which involves Gaussian distributions.
    - To better model real-world shape distributions, which may be arbitrarily complex, Gaussian kernel space density estimation (Cremers et al. 2003) and, more recently, Parzen kernel density estimation (Cremers et al. 2006a) were proposed.

Introduction[3]

- Relationship to prior work
  - The second issues: alignment
    - When using reference shape, it must be also dealt with a question about shape alignment.
    - Pose parameters (rotation, translation and scaling) are generally taken into account in an explicit fashion. But this leads the system to be complicated and coupled partial differential equations.
    - To overcome the problem, implicit representation is then proposed in:
      - Székely et al. (1996), for explicit snakes implementations
      - Cremers et al.(2006a), Foulonneau et al. (2003) for implicit representations in the case of translation and scale invariance
      - Foulonneau et al. (2006a) for the affine invariance

Introduction[4]

- Contribution of our work
  - The approach reported in the present paper combines a compact parametric representation of shapes with curve evolution theory.
  - The advantage of our model is that it is not bound to any particular implementation. Level sets (Osher and Sethian 1988) or spline-snakes (Precioso and Barlaud 2002) may be considered
  - More specifically, this parametric description is based on Legendre moments which can make our shape descriptor intrinsically affine-invariant. This avoids the problem of pose estimation.

Introduction[5]

- Contribution of our work [cont.]
  - Among the invariant models previously proposed,
    - Székely et al. (1996), Cremers et al. (2002) cannot handle complex topologies
    - Cremers et al. (2006a), Foulonneau et al. (2003) can handle the complex but are limited to translation and scale invariance.
    - Riklin-Raviv et al. (2004) can handle projective invariance but use an explicit formulation
    - Finally, the pose parameters are readily available by our method.
  - We address the multi-reference case, i.e. multiple reference shapes are *simultaneously considered.*

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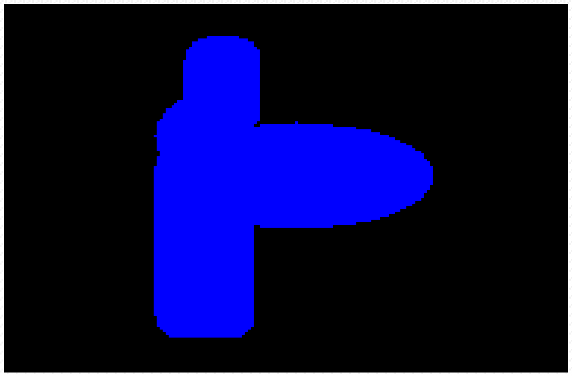
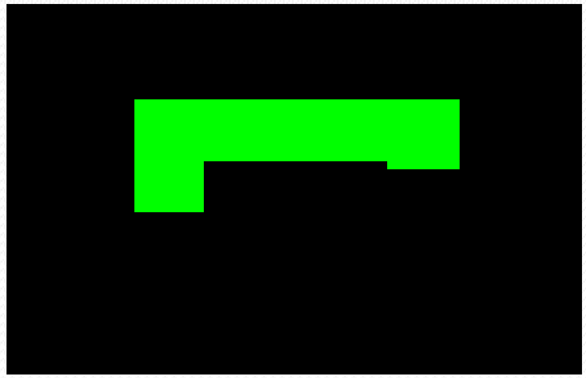
Shape Descriptor and Geometrical Invariance

- We represent a shape by a shape descriptor (order=N).
- The shape descriptor is defined as the *D-dimensional vector* of Legendre moments, *where* $D = (N + 1)(N + 2)/2$.





Watch 1

Name	Value
humoments	0x01515f70 { hu1=0.4593781290205
hu1	0.45937812902051489
hu2	0.16655638756181437
hu3	0.0073864757504644839
hu4	0.0023001370569503406
hu5	0.0000081676553071631067
hu6	0.00069719583547974936
hu7	-0.0000048142210662061

Watch 1

Name	Value
humoments	0x015125e8 { hu1=0.22586190025149
hu1	0.22586190025149455
hu2	0.0017015233948888262
hu3	0.0071204325618661971
hu4	0.000078273412836902753
hu5	-0.000000051723137170647175
hu6	-0.0000027253728182335237
hu7	-0.000000027191737910398332

Shape Descriptor and Geometrical Invariance

- Encoding shape with Legendre moments
  - (p+q) is called the order of the moment *N*
  - Legendre moment:

$$\lambda_{p,q} = C_{pq} \sum_{u=0}^{p} \sum_{v=0}^{q} a_{pu} a_{qv} M_{u,v}.$$

  - Normalizing Constance: $C_{pq} = (2p+1)(2q+1)/4$
  - Regular moment : $M_{p,q} = \iint f(x,y) x^p y^q \, dx dy$
  - Characteristic function : $f(x,y)$
  - Pixel coordinate : $(x,y) \in [-1,1]^2$

Shape Descriptor and Geometrical Invariance

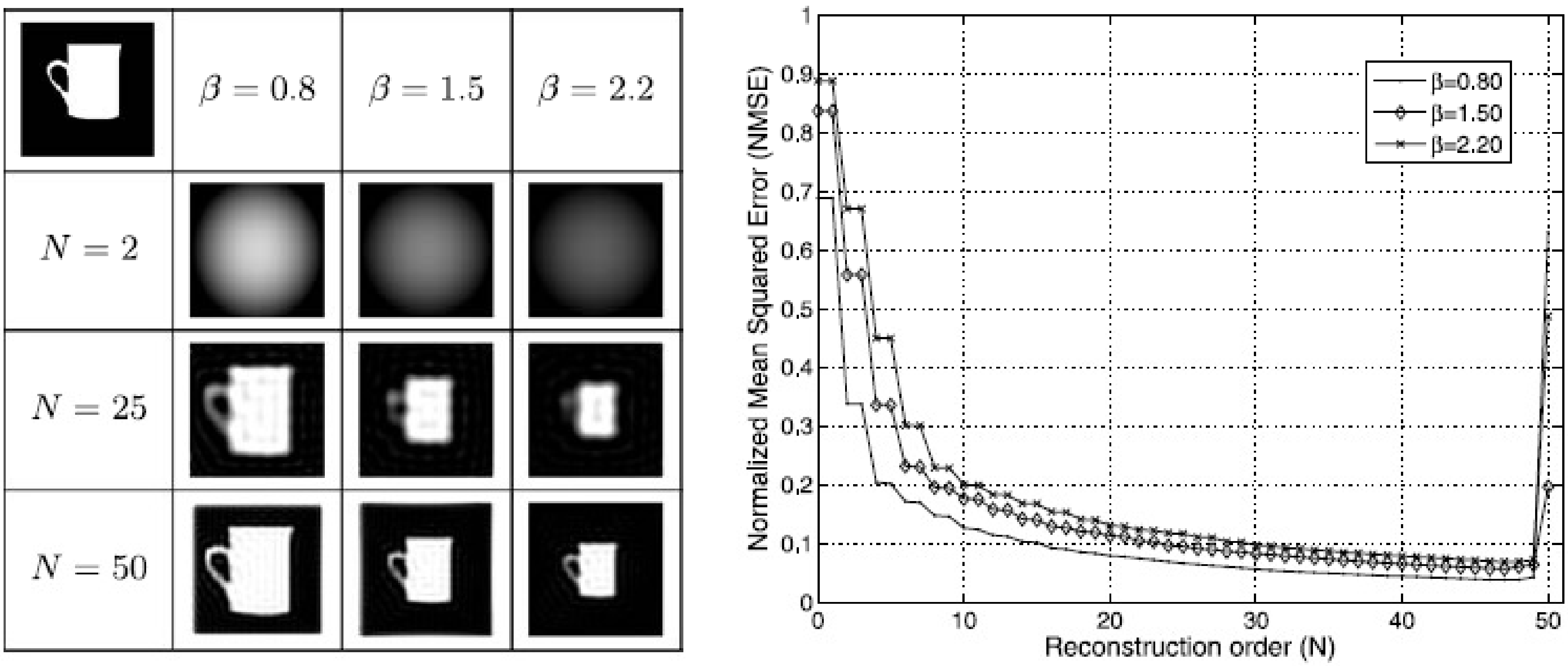
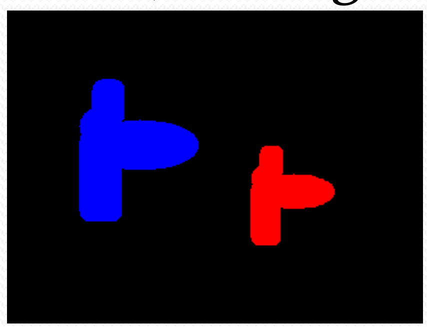


Fig. 1 Reconstructions of a mug shape from its Legendre moments made invariant w.r.t. scale and translation (*left*) and corresponding reconstruction error curves (*right*)

Shape Descriptor and Geometrical Invariance

- <u>Handling Pose and Geometric Variability</u>
  - Two shapes (same object but transformed) have the same descriptors (Legendre moment)
  - Example of Translation, Scaling-invariance:



Watch 1

Name	Value
⊟ humoments	0x01512730 { hu1=0.227051772698
hu1	0.22705177269881588
hu2	0.0017841338858215396
hu3	0.0072459568273482294
hu4	0.000090501212202107943
hu5	-0.00000000655012872510 7443
hu6	-0.000003212602201101 6145
hu7	-0.00000000328729083064 03577

Watch 1

Name	Value
⊟ humoments	0x01512878 { hu1=0.2258619002514
hu1	0.22586190025149455
hu2	0.0017015233948888262
hu3	0.0071204325618661971
hu4	0.000078273412836902753
hu5	-0.0000000051723137170647175
hu6	-0.0000027253728182335237
hu7	-0.0000000027191737910398332

Shape Descriptor and Geometrical Invariance

- <u>Handling Pose and Geometric Variability</u>
  - Two shapes (same object but transform) have the same descriptors (Legendre moment)
  - Affine transformation includes translation, scaling and rotation.
  - This solves the alignment problem and makes the model invariant w.r.t. the geometrical transformation.
  - This introduces geometrical variability in the model.
  - Descriptor can be invariance by replacing $M_{u,v}$ in Legendre moment by proper expression.

$$\lambda_{p,q} = C_{pq} \sum_{u=0}^{p} \sum_{v=0}^{q} a_{pu} a_{qv} M_{u,v}.$$

Shape Descriptor and Geometrical Invariance

- Handling Pose and Geometric Variability
  - *Scale and Translation Invariance* (λ)
    - This invariance descriptor can be obtain by aligning the shape centroid $(\overline{x}, \overline{y})$ with center of the domain and normalize its area to a constant $1/\beta$.

$$\lambda_{p,q} = C_{pq} \sum_{u=0}^{p} \sum_{v=0}^{q} a_{pu} a_{qv} \eta_{u,v}$$

$$\eta_{u,v} = \iint_{\Omega_{in}} \frac{(x-\overline{x})^u (y-\overline{y})^v}{(\beta|\Omega_{in}|)^{(u+v+2)/2}} dxdy$$

    - This uses *normalized central moment* ($\eta_{u,v}$) *instead of* $M_{u,v}$

Shape Descriptor and Geometrical Invariance

- Handling Pose and Geometric Variability
  - *Affine Invariance* (λ^A)
    - This invariance descriptor can be obtain by *image normalization procedure* (Pei and Lin 1995).

$$\lambda^A_{p,q}(\Omega_{in}) = C_{pq} \sum_{u,v}^{u \le p, v \le q} a_{pu} a_{qv} \eta^A_{u,v}$$

$$\eta^A_{u,v} = \left(\text{sign}\left(\widehat{\eta}^A_{3,0}\right)\right)^u \cdot \left(\text{sign}\left(\widehat{\eta}^A_{0,3}\right)\right)^v . \widehat{\eta}^A_{u,v}$$

$$\widehat{\eta}^A_{u,v} = \frac{(l_1 \cdot l_2)^{\frac{u+v}{4}}}{(\beta|\Omega_{in}|)^{(u+v+2)/2}} \times \iint_{\Omega_{in}} \left(\frac{((x-\overline{x})\cos\theta + (y-\overline{y})\sin\theta)}{\sqrt{l_1}} \cos\gamma + \frac{((y-\overline{y})\cos\theta - (x-\overline{x})\sin\theta)}{\sqrt{l_2}} \sin\gamma \right)^u$$

$$\times \left(\frac{((y-\overline{y})\cos\theta - (x-\overline{x})\sin\theta)}{\sqrt{l_2}} \cos\gamma - \frac{((x-\overline{x})\cos\theta + (y-\overline{y})\sin\theta)}{\sqrt{l_1}} \sin\gamma \right)^v dx dy.$$

Shape Descriptor and Geometrical Invariance

- Handling Pose and Geometric Variability
  - *Similarity Invariance (λ^S)*
    - This invariance descriptor can be obtain by setting γ=0 and l1, l2 =1.

$$\lambda^S_{p,q}(\Omega_{in}) = C_{pq} \sum_{u,v}^{u \le p, v \le q} a_{pu} a_{qv} \eta^S_{u,v}$$

$$\eta^S_{u,v} = \left(\text{sign}\left(\widehat{\eta}^S_{3,0}\right)\right)^u \cdot \left(\text{sign}\left(\widehat{\eta}^S_{0,3}\right)\right)^v \cdot \widehat{\eta}^S_{u,v},$$

and

$$\widehat{\eta}^S_{u,v} = \iint_{\Omega_{in}} \frac{((x-\overline{x})\cos\theta + (y-\overline{y})\sin\theta)^u}{(\beta|\Omega_{in}|)^{(u+v+2)/2}} \times ((y-\overline{y})\cos\theta - (x-\overline{x})\sin\theta)^v dxdy$$

Shape Descriptor and Geometrical Invariance

Fig. 2 Reconstruction of shapes from their affine-invariant moments. *Upper row*: original shapes. *Lower row*: shape reconstructions using affine invariant moments (these reconstruction correspond to the canonical representations of the original shapes)

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Multi-Reference Shape Priors

Fig. 4 Set of reference shapes used with the multi-reference model (19)

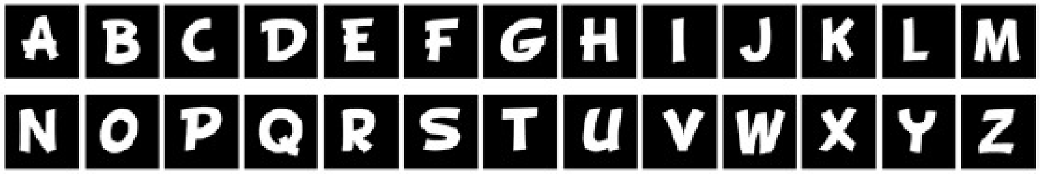
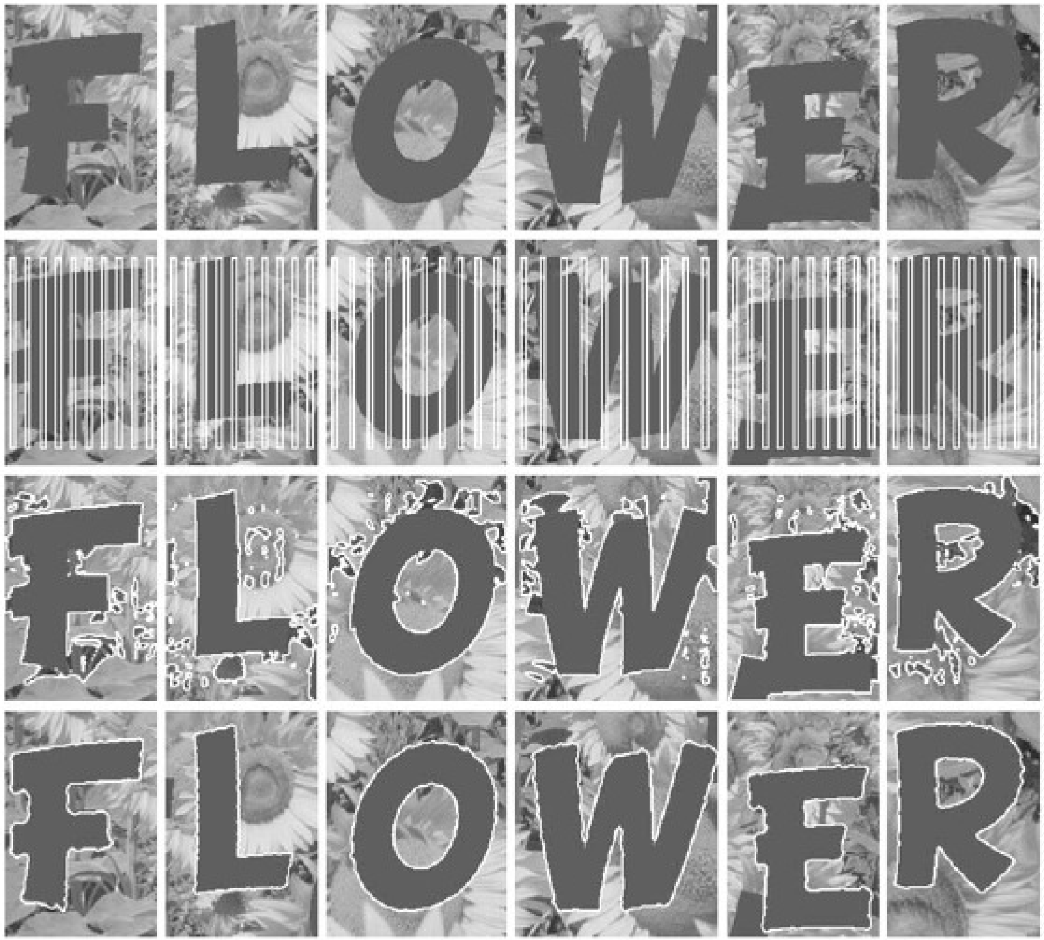


Fig. 5 Segmentation of synthetic images. *First row*: initial image, *second row*: initial contours, *third row*: segmentation results without shape prior, *fourth row*: segmentation result using the multi-reference prior invariant to scaling and translation: moments up to the 40th-order ($\beta = 1.5$)



Multi-Reference Shape Priors

- Evolution Equation for the Multi-Reference Model

Depending on the choice of the descriptor $\boldsymbol{\lambda}, \boldsymbol{\lambda}^S$ or $\boldsymbol{\lambda}^A$, the proposed constraint is able to handle several levels of geometric invariance,

$$\frac{\partial \Gamma}{\partial t} = \sum_{u,v}^{u+v \leq N} A_{uv}^{multi} \left(H_{uv}(x, y, \Omega_{in}) + \sum_{i=0}^{2} B_{uvi} \cdot L_i(x, y) \right) \mathcal{N},$$

$$A_{uv}^{multi} = \frac{1}{2\sigma^2 \sum_{k=1}^{N_{ref}} \exp(\frac{-\|\lambda - \lambda_{(k)}^{ref}\|^2}{2\sigma^2})} \times \sum_{k=1}^{N_{ref}} A_{(k)uv} \exp\left(\frac{-\|\lambda - \lambda_{(k)}^{ref}\|^2}{2\sigma^2} \right)$$

$$H_{uv}(x, y, \Omega_{in}) = \frac{(x - \overline{x})^u (y - \overline{y})^v}{(\beta |\Omega_{in}|)^{(u+v+2)/2}}, \qquad B_{uv0} = \frac{u \cdot \overline{x} \cdot \eta_{u-1,v} + v \cdot \overline{y} \cdot \eta_{u,v-1}}{\beta^{\frac{1}{2}} |\Omega_{in}|^{\frac{3}{2}}} - \frac{(u+v+2) \cdot \eta_{u,v}}{2|\Omega_{in}|}$$

$$B_{uv1} = \frac{-u \cdot \eta_{u-1,v}}{\beta^{\frac{1}{2}} |\Omega_{in}|^{\frac{3}{2}}}, \qquad B_{uv2} = \frac{-v \cdot \eta_{u,v-1}}{\beta^{\frac{1}{2}} |\Omega_{in}|^{\frac{3}{2}}}, \qquad L_0 = 1, \qquad L_1 = x, \qquad L_2 = y.$$

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Application to Image Segmentation

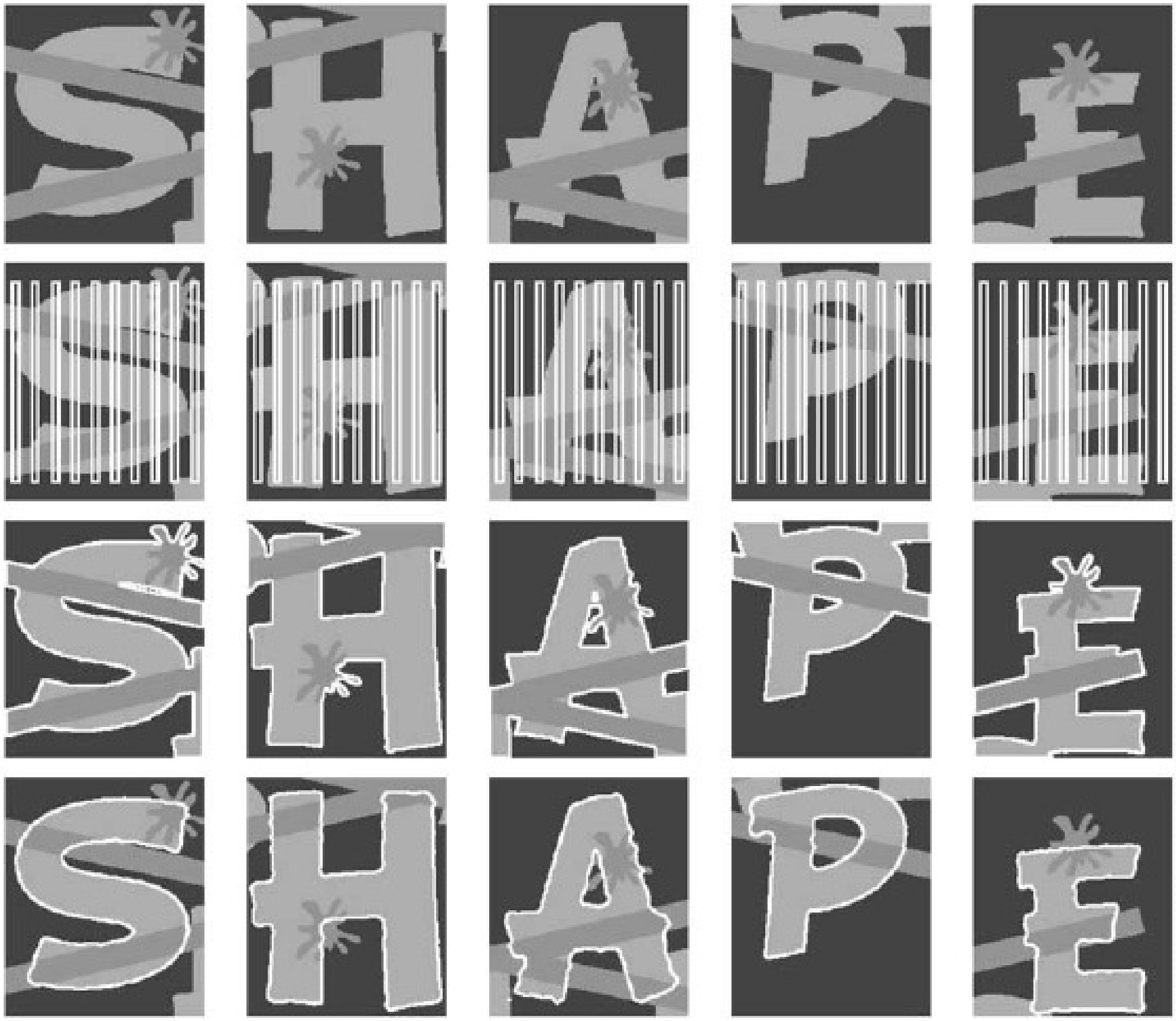


Fig. 6 Segmentation of five images of letters featuring large occlusions. *First row*: original images. *Second row*: initialization. *Third row*: results without shape constraint (no standard curvature component). *Fourth row*: final results, adding the multi-reference prior invariant to scaling and translation, up to the order 40 ($\beta = 1.5$). The same set of parameters is used for all the experiments

Application to Image Segmentation

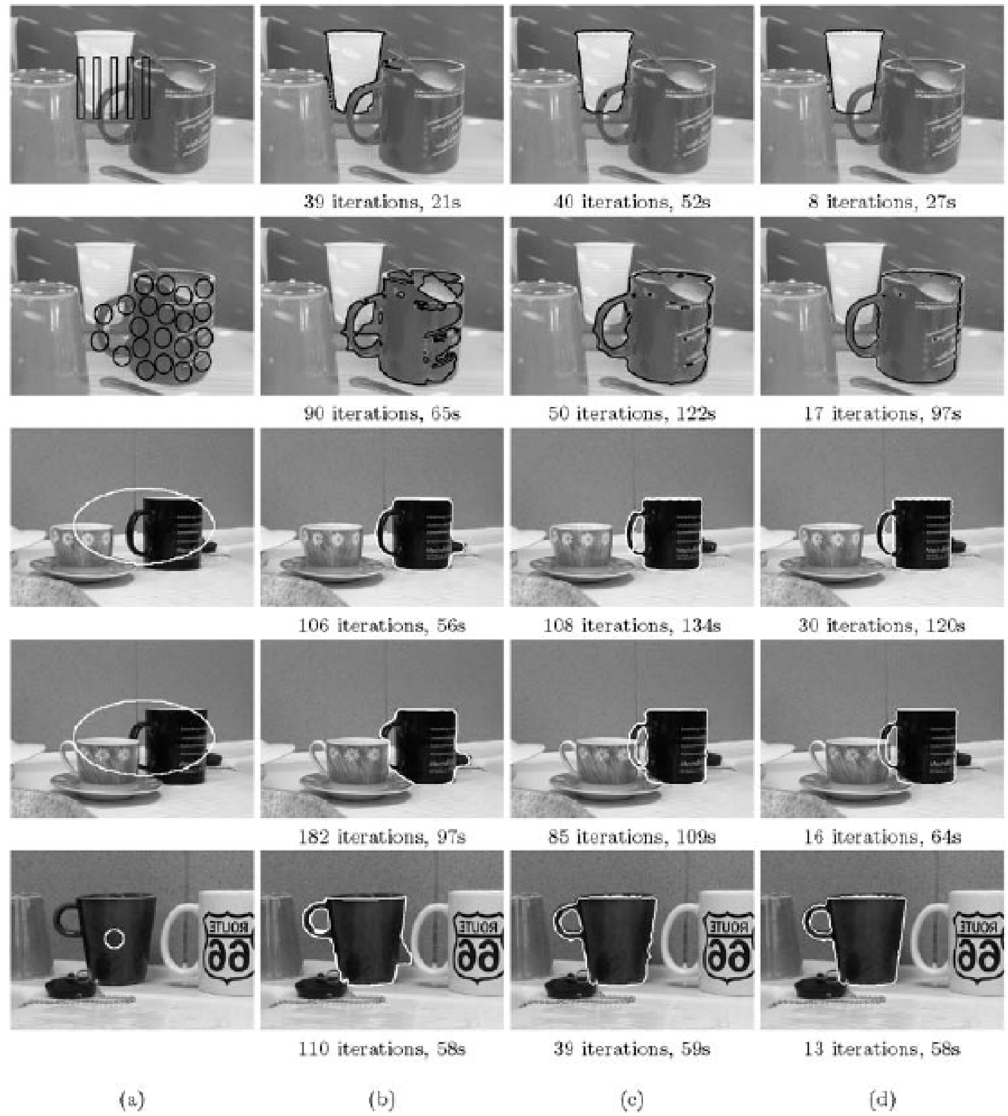


Fig. 8 Segmentation of real images. (**a**) Initial contours, (**b**) segmentation results without shape prior (standard curvature component used), segmentation result using the multi-reference prior invariant to scaling and translation: (**c**) moments up to the 20th-order and (**d**) to the 40th-order ($\beta = 0.8$). The number of evolution iterations performed at each stage and the corresponding computation time are given underneath each result

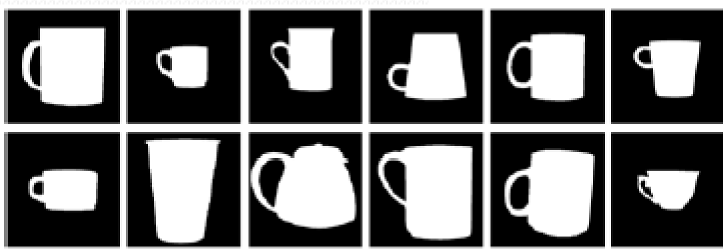


Fig. 7 Set of reference shapes used for the experience of Fig. 8

Application to Image Segmentation

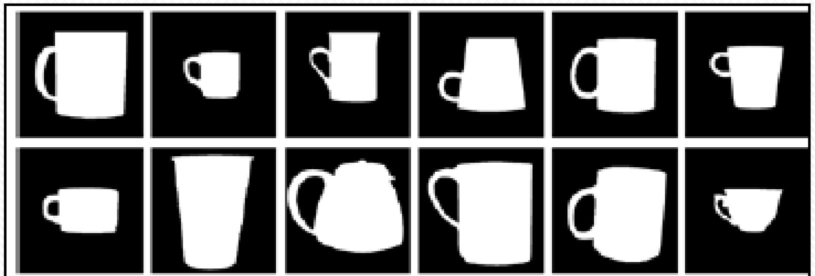


Fig. 7 Set of reference shapes used for the experience of Fig. 8

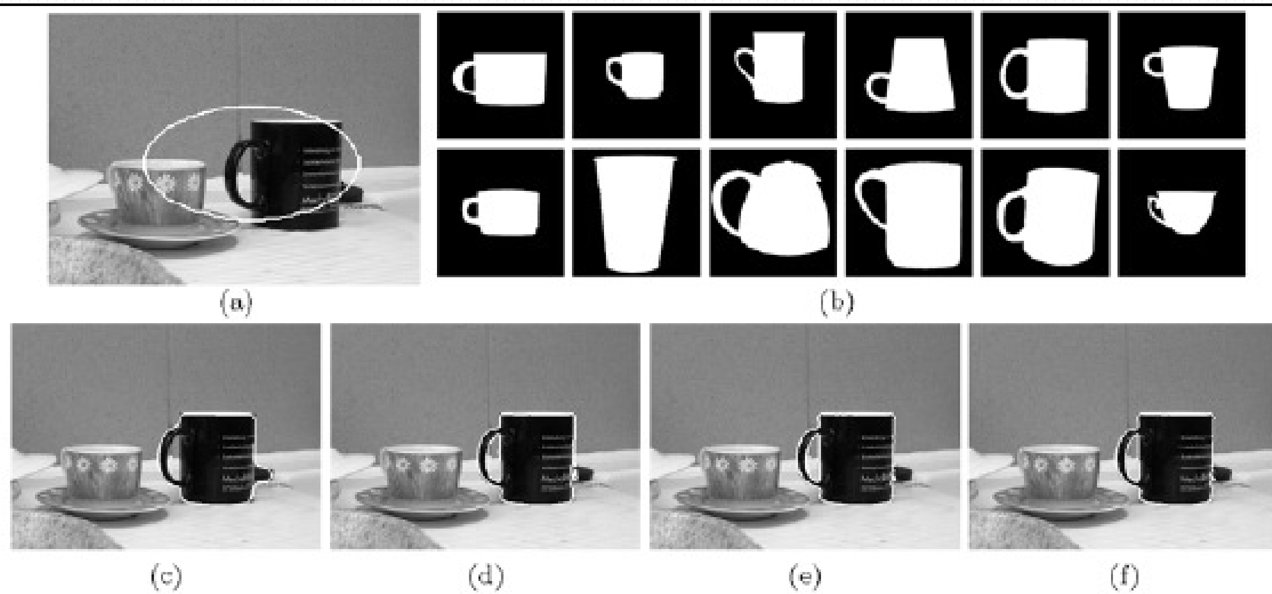


Fig. 9 Segmentation of a real image. (a) Initial curve; (b) set of reference images; (c) segmentation without shape constraint (standard curvature component used); segmentation result using the multi-reference, affine-invariant shape prior: (d) moments up to the 18th order, (e) to the 30th order and (f) to the 40th order

Application to Image Segmentation

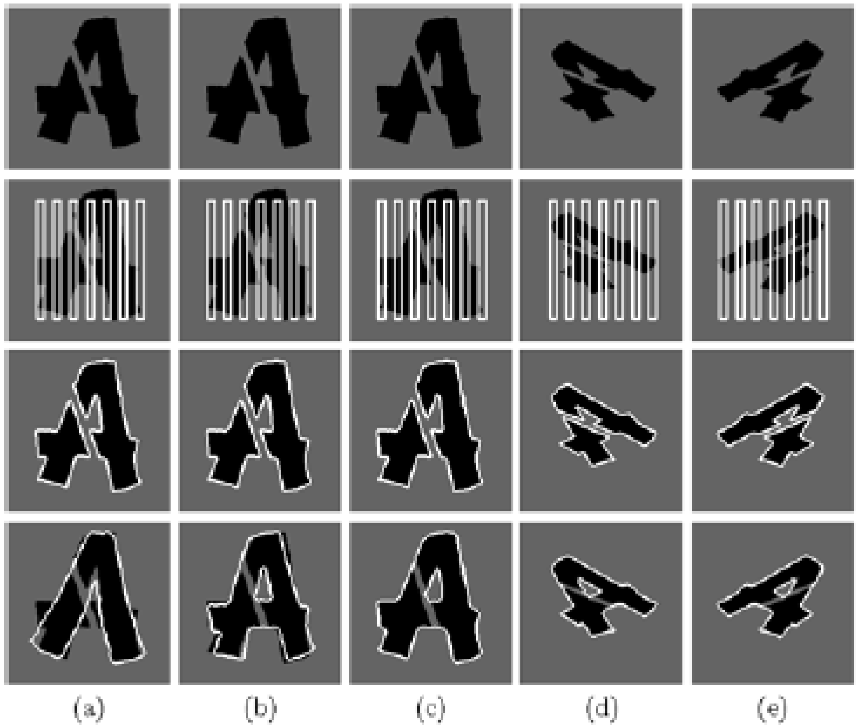
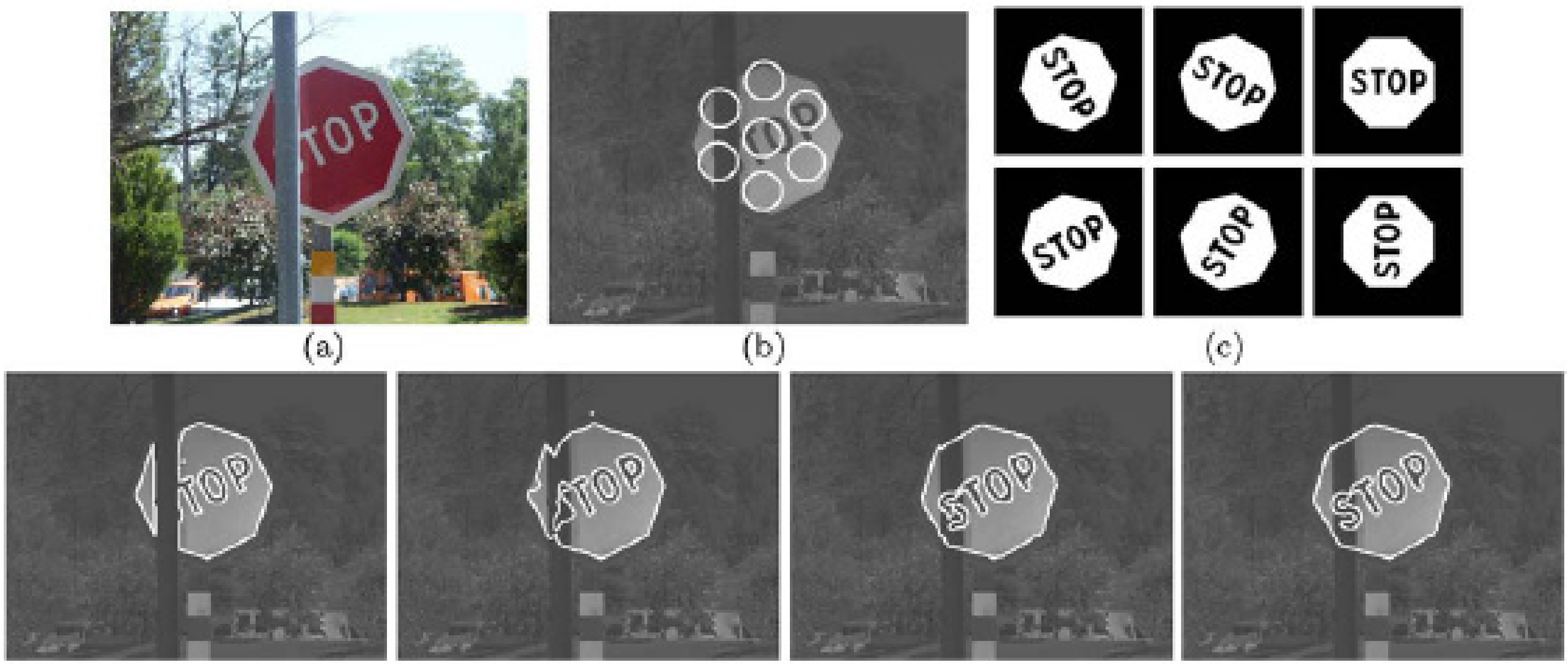


Fig. 10 Segmentation of synthetic images with the multi-reference prior using different levels of invariance and different set of reference shapes. *First line*: original images; *second line*: initial curves; *third line*: segmentation results without any prior; *fourth line*: final segmentation results ($N = 45$). **(a)** The multi-reference shape prior is affine-invariant and simultaneously takes into account the 26 letters of the alphabet presented in Fig. 4. **(b)** The same set of reference shapes is used but invariance is limited to translation and scale. **(c)**, **(d)**, **(e)** The multi-reference shape prior is affine-invariant but the set of reference shapes does not include the "V" letter

Application to Image Segmentation



(a) (b) (c)
(d) (e) (f) (g)

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Conclusion

- In this paper, we have presented a novel approach for the integration of multiple prior shape models in active contour based image segmentation.
- Our multi-reference prior shape model relies on affine-invariant shape descriptors related to Legendre moments.
- A unique evolution equation for the active contour is derived, using the formalism of shape derivative.
- Experimental results shows that the proposed evolution equation introduce noticeable robustness to background clutter and occlusions.
- Some issues remain open.
  - Choice of adequate initialization converging toward the desired solution
  - The approach enables only one single shape at the same time (even if multiple reference are taken into account)

THANK YOU