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An Axiomatic Analysis of Structured Argumentation for Prioritized Default Reasoning

Phan Minh Dung [1]

Abstract. Several systems of argument-based and non-argument-based semantics have been proposed for prioritized default reasoning. As the proposed semantics often sanction contradictory conclusions (even for skeptical reasoners), there is a fundamental need for guidelines for understanding and evaluating them, especially their conceptual foundations and relationships. In this paper, we introduce several natural axioms for structural argumentation with preferences that capture both the consistency and closure postulates. We show that Aspic$^+$ semantics do not satisfy key axioms including the consistency postulate and propose a simple one satisfying all axioms. We show that the prescriptive non-argument-based approach to prioritized default reasoning is sound (and complete for a relevant class of knowledge bases) wrt our proposed simple semantics.

1 Introduction

The diversity of distinct semantics proposed for structured argumentation with preferences [15, 14] or prioritized default reasoning [16, 2, 4, 18, 17, 7, 10, 11] that often sanction contradicting conclusions (even for skeptical reasoner), raises a fundamental question of how to characterize and evaluate these semantics when an user applies prioritized default reasoning in reality.

It was persuasively argued in [3] that *the intuition of default reasoning* is about finding (justified) belief sets that give *the most accurate picture of reality* assuming the world is as normal as possible. In this paper, we present a set of axioms to capture this *reality-representing-intuition* and apply them to evaluate the semantics of structured argumentation.

Naturally, if a (justified) belief set is supposed to represent an accurate picture of reality then the confirmation by reality of some beliefs in this set obviously does not invalidate other beliefs in it. In contrary, it should strengthen our conviction that the world is indeed captured by this belief set. We represent this aspect of the reality representing intuition by the axioms of credulous cumulativity generalizing the skeptical cumulative property of nonmonotonic inference [8, 13] and attack monotonicity whose intuition is that the more hard evidences your arguments are based on, the stronger your arguments become and hence the more arguments are attacked by them. It turns out that the credulous cumulativity axiom is stronger than the consistency postulate [1] in the sense that the later follows from the former but not vice versa. Somewhat surprisingly, the axiom of credulous cumulativity is not satisfied by key semantics of Aspic$^+$ based on the elitist preorder. A simple example is given below (A more involved example 5.1 shows that these semantics do not satisfy the consistency postulate).

Example 1.1 *Consider a knowledge base K (adapted from [7]), consisting of three defeasible rules $d_1 : D \Rightarrow P,\ d_2 : P \Rightarrow T,\ d_3 : A \Rightarrow \neg T,$[2] and one strict rule $D \rightarrow A$ where $d_1 \prec d_3 \prec d_2$ and $d_i \preceq d_i$, $i = 1..3$.[3] Suppose we know some Dean. Consider three arguments: $A_1 = [D \Rightarrow P],\ A_2 = [D \Rightarrow P \Rightarrow T],\ A_3 = [D \rightarrow A \Rightarrow \neg T]$.*

According to the weakest link principle and the elitist preordering in Aspic+, A_3 attacks A_2 but not vice versa.[4] There is accordingly an unique stable extension containing A_1, A_3 concluding that the dean is a professor but not teach. Suppose that it turns out that the dean is indeed a professor resulting in a new knowledge base $K' = K + \{P\}$ with a new argument $A'_2 = [P \Rightarrow T]$ that is preferred to A_3. Therefore the unique stable extension of K' contains A_1, A'_2, A_2 implying that the dean does teach. The semantics violates the cumulativity property. □

Imagine you have a lively dancing bird in your garden and you know that it is a penguin. Suppose some neighbour tells you that the bird is most likely a penguin. Will it change anything in your beliefs about your bird ? Of course not. This is an example of the property of irrelevance of redundant defaults stating that adding redundant defaults into your knowledge base does not change your beliefs. Simple and natural as it is, this property is surprisingly not satisfied by the semantics of Aspic$^+$ that are based on democratic order as illustrated below.

Example 1.2 *Consider a knowledge base consisting of two defeasible rules $d_1 : \Rightarrow a_1$, $d_2 : \Rightarrow a_2$ and three strict rules $r_1 : b, a_1 \rightarrow \neg a_2$, $r_2 : b, a_2 \rightarrow \neg a_1$, $r_3 : a_1, a_2 \rightarrow \neg b$ such that $d_1 \prec d_2$ and $d_i \preceq d_i$, $i = 1, 2$. Further, let b be the only evidence representing an unchallenged observation of reality. Consider the arguments $A_1 = [\Rightarrow a_1],\ A_2 = [\Rightarrow a_2],\ B = [b],\ N_1 = [B, A_1 \rightarrow \neg a_2],\ N_2 = [B, A_2 \rightarrow \neg a_1]$ and $N = [A_1, A_2 \rightarrow \neg b]$. Due to the preference of d_2 over d_1, N_2 attacks A_1 but N_1 does not attack A_2 wrt all four attack relations in Aspic$^+$. Therefore N_2 also attacks N_1, N. The unique stable extension is thus $\{A_2, B, N_2\}$. Hence $\neg a_1, a_2$ are skeptically justified.*

Suppose for whatever reason, we add a defeasible rule $d : \Rightarrow b$ into the knowledge base. As b is an unchallenged evidence, we expect that the semantics of the knowledge base does not change at all. But surprisingly Aspic$^+$ semantics, based on democratic order, sanction new justified beliefs that are contrary to the original one as follows: Let $B' = [\Rightarrow b]$, $N'_1 = [B', A_1 \rightarrow \neg a_2]$, $N'_2 = [B', A_2 \rightarrow \neg a_1]$. According to the Aspic$^+$ attack relations based on the democratic

[1] Asian Institute of Technologies, Thailand, email: dung.phanminh@gmail.com

[2] D,P,T,A stand for Dean, Professor, Teach and Administrator respectively.
[3] $x \preceq y$ means x is less or equally preferred than y.
[4] In [14, 15], attack was referred to as defeat.

order, N'_1 attacks A_2. Hence N'_1 also attacks N_2, N'_2, N. Hence $\{A_1, B, B', N'_1\}$ is also a stable extension justifying $a_1, \neg a_2$. □

In this paper, we present several simple and natural axioms formalizing different aspects of the reality-representing intuition. As the proposed semantics in Aspic$^+$ do not satisfy key axioms like consistency, cumulativity or attack monotonicity, we propose a novel new attack relation referred to as normal attack relation, that satisfies all axioms and is not based on preferences between arguments. We show that the prescriptive non-argument-based approach in [4, 18] that is based on an operational view of rule preferences, is sound (and complete for a relevant class of knowledge bases) wrt normal semantics of prioritized default reasoning.

2 Preliminaries

An abstract argumentation framework [5] is defined simply as a pair (AR, att) where AR is a set of arguments and $att \subseteq AR \times AR$. $(A, B) \in att$ means that A *attacks* B. A set of argument S attacks (or is attacked by) an argument A if some argument in S attacks (or is attacked by) A; S is *conflict-free* if it does not attack itself. A set of arguments S defends an argument A if S attacks each attack against A. S is *admissible* if S is conflict-free and defends each argument in it. The semantics of abstract argumentation is defined by various notions of extensions. A *complete extension* is an admissible set of arguments containing each argument it defends. A *stable extension* is a conflict-free set of arguments that attacks every argument not belonging to it. It is well-known that stable extensions are complete but not vice versa.

3 Prioritized Knowledge Bases

We assume a set $\mathcal{L}$ of ground atoms and their classical negations. An atom is also called a positive literal while a negative literal is the negation of a positive literal. A set of literals is said to be **contradictory** if it contains a pair $l, \neg l$.

A *strict/defeasible rule* r is of the form $b_1, \ldots, b_n \rightarrow / \Rightarrow h$ respectively where $b_1, \ldots, b_n, h$ are literals from $\mathcal{L}$. The set $\{b_1, \ldots, b_n\}$ (resp. the literal h) is referred to as the body (resp. head) of r, denoted by $bd(r)$ (resp. $hd(r)$). For a set of rules R, denote $hd(R) = \{hd(r) \mid r \in R\}$.

A **rule-based system** is defined as a triple $RBS = (RS, RD, \preceq)$ of a set RS of strict rules, a finite set RD of defeasible rules and a preorder (i.e. a reflexive and transitive relation) $\preceq$ over RD. We write $\mathbf{d} \prec \mathbf{d'}$ iff $d \preceq d'$ and $d' \npreceq d$.

For each defeasible rule $d \in RD$, there is a distinct atom $\mathbf{ab_d}$ that does not occur in the bodies of any rules in $RS \cup RD$ but could appear in the heads of some strict rules of the form $bd \rightarrow ab_d$ stating that d must not be used when bd hold. Literals whose atoms are not of the form ab_d are called **domain literals**.

A **base of evidence** BE is a finite set of ground domain literals representing unchallenged observations, facts ect..

Definition 3.1 *A* **knowledge base** *is a pair (RBS, BE) of a rule-based system $RBS = (RS, RD, \preceq)$ and a base of evidences BE.*

Definition 3.2 *Let $K = (RBS, BE)$ be a knowledge base. An argument wrt K is defined as follows:*

1. *For each $\alpha \in BE$, $[\alpha]$ is an argument with conclusion α.*
2. *Let r be a rule of the forms $\alpha_1, \ldots, \alpha_n \rightarrow / \Rightarrow \alpha$, $n \geq 0$, from RBS. Further suppose that $A_1, \ldots, A_n$ are arguments with conclusions α_i, $1 \leq i \leq n$, respectively. Then $A = [A_1, \ldots, A_n \rightarrow / \Rightarrow \alpha]$ is an argument with conclusion α and last rule r denoted by* cnl(A) *and* last(A) *respectively. For simplicity, A is often denoted by $[A_1, \ldots, A_n, r]$.* □

Notation 1 *1. The set of all arguments wrt a knowledge base K is denoted by $\mathbf{AR_K}$. The set of conclusions of arguments in a set S is denoted by $\mathbf{cnl(S)}$.*

2. A **strict** *argument is an argument containing no defeasible rule. An argument is* **defeasible** *iff it is not strict. A defeasible argument A is called* **basic defeasible** *iff $last(A)$ is defeasible.*

3. An argument B is a **subargument** *of an argument $A = [A_1, \ldots, A_n, r]$ iff $B = A$ or B is a subargument of some A_i.*

Definition 3.3 *Let $A, B \in AR_K$ for a knowledge base K.*

A **rebuts** *B (at B') iff B' is a basic defeasible subargument of B such that the conclusions of A and B' are contradictory.*

A **undercuts** *B (at B') iff B' is a basic defeasible subargument of B such that the conclusion of A is $ab_{last(B')}$.* □

Notation 2 *1. The set of defeasible rules appearing in an argument B is denoted by $\mathbf{dr(B)}$. For an argument $A = [A_1, \ldots, A_n, r]$, the set of defeasible rules appearing last in A, denoted by $\mathbf{ldr(A)}$, is defined by: $ldr(A) = \{r\}$ if r defeasible, otherwise $ldr(A) = ldr(A_1) \cup \ldots \cup ldr(A_n)$.*

2. The **closure** *of a set of ground literals X, denoted by $CN_{RS}(X)$ is the set of conclusions of all strict arguments over the set of strict rules RS with X as the base of evidence.*
We often write $CN(X)$ or $CN_K(X)$ for $CN_{RS}(X)$. We also often write $\mathbf{X \vdash l}$ (or $\mathbf{X \vdash_K l}$) if $l \in CN(X)$.

3. X is said to be **closed** *iff $X = CN(X)$. X is said to be* **inconsistent** *iff its closure $CN(X)$ is contradictory. X is* **consistent** *iff it is not inconsistent.*

A knowledge base K is said to be *consistent* iff its base of evidence is consistent wrt its set of strict rules.

K is said to be *closed under transposition* iff for each strict rule of the form $b_1, \ldots, b_n \rightarrow h$ in K s.t. h is a domain literal, all the rules $b_1, \ldots, b_{i-1}, \neg h, b_{i+1}, \ldots, b_n \rightarrow \neg b_i$, $1 \leq i \leq n$, also belong to K.

K is said to be *closed under contraposition* iff for each set of domain literals S, each domain literal λ, if $S \vdash_K \lambda$ then for each $\sigma \in S$, $S \setminus \{\sigma\} \cup \{\neg\lambda\} \vdash_K \neg\sigma$.

K is said to satisfy the *self-contradiction property* iff for each minimal inconsistent set of domain literals $X \subseteq \mathcal{L}$, for each $x \in X$, it holds: $X \vdash_K \neg x$.

The following lemma is proved in [6].

Lemma 3.1 *If K is closed under transposition or contraposition then K satisfies the self-contradiction property.* □

4 Introducing New Axioms

Notation 3 • *From now until the end of this chapter, we assume that an attack relation $att(KB) \subseteq AR_{KB} \times AR_{KB}$ has been defined for each knowledge base KB.*

- *Abusing the notation, we often refer to* att *as* attack relation *and refer to the argumentation framework $(AR_K, att(K))$ often simply as (AR_K, att)*
- *We assume an arbitrary but fixed consistent knowledge base $K = (RBS, BE)$ with $RBS = (RD, RS, \preceq)$.*
- *For any finite set Ω of domain literals, define $K + \Omega = (RBS, BE \cup \Omega)$.* □

Definition 4.1 *A set $S \subseteq \mathcal{L}$ is said to be a* stable *(resp* complete*)* belief set *of K if there is a stable (resp complete) extension E of $(AR_K, att(K))$ such that $S = cnl(E)$.*

We start with the introduction of axiom of effective rebut stating that the purpose of preference between rules is to render some rebuts ineffective.

Example 4.1 *Suppose K consists of just two defeasible rules: $d_1 : \Rightarrow a$, $d_2 : \Rightarrow \neg a$, with $d_1 \prec d_2$. The arguments $A_1 = [\Rightarrow a]$, $A_2 = [\Rightarrow \neg a]$ rebut each other. As d_2 is preferred to d_1, A_2 is an effective rebut against A_1 while the reverse does not hold. Hence A_2 is an attack against A_1, but not vice versa.* □

Definition 4.2 *(***Effective Rebut***) We say attack relation* att *satisfies the* effective rebut axiom *if for all arguments A_1, A_2 such that A_2 rebuts A_1 and each A_i, i = 1,2, contains exactly one defeasible rule d_i, it holds that A_2* attacks *A_1* iff *$d_2 \not\preceq d_1$.* □

The defeasible sub-arguments of an argument form the defeasible points at which attacks could be launched against the argument.

Definition 4.3 *(***Subargument Structure***) We say attack relation* att *satisfies the* axiom of subargument structure *iff for all arguments $A, B \in AR_K$, A attacks B iff A attacks a defeasible subargument of B (wrt att(K)).*

It is straightforward to see

Lemma 4.1 *Suppose attach relation* att *satisfies the axiom of subargument structure. Then* att *also satisfies the closure axiom, i.e. for each complete extension E of K, $cnl(E)$ is closed.*

Structured argumentation systems [14, 1] employ attack relations that are context-independent.

Definition 4.4 *(***Context-Independence***) We say attack relation* att *satisfies* the axiom of context-independence *iff for any two arbitrary knowledge bases K, K' with preference preorders $\preceq, \preceq'$ respectively and any two arguments A, B belonging to $AR_K \cap AR_{K'}$ such that the restriction of $\preceq$ and $\preceq'$ on $dr(A) \cup dr(B)$ coincide, it holds that*

$(A, B) \in att(K)$ *iff* $(A, B) \in att(K')$.

We proceed with the formalization of the property of irrelevance of redundant defaults.

Definition 4.5 *(***Irrelevance of Redundant Defaults***) We say attack relation* att *satisfies the property of* irrelevance of redundant defaults *iff for each evidence $\omega \in BE$, the stable belief sets of K and K' coincide where $K' = K + d = (RD \cup \{d\}, RS, \preceq', BE)$ and d is the defeasible rule $\Rightarrow \omega$ and $\preceq' = \preceq \cup \{(d, d)\}$.* □

The property of irrelevance of redundant defaults follows from the the axiom of attack monotonicity stating that when some piece of defeasible information on which an argument is based is confirmed by an unchallenged observation, the argument is strengthened in the sense that whatever is attacked by the original argument should also be attacked by the strengthened one.

Example 4.2 *(Continuing example 1.2)*

Let us look at example 1.2 again. N_1 could be obtained from N_1' by replacing the defeasible rule $\Rightarrow b$ by the hard evidence b. Hence N_1 should be stronger than N_1'. Therefore if N_1' attacks A_2, we expect N_1 also attacks A_2 what is not the case according to the attack relations based on democratic order in Aspic$^+$. □

Let $A \in AR_K$ and $S \subseteq BE$ be a finite set of literals. The **strengthening** of A wrt S denoted by $\mathbf{A \uparrow S}$, is the set of arguments obtained by replacing zero, one or more subarguments of A by their conclusions provided that these conclusions belong to S. For illustration, in example 1.2, $B' \uparrow \{b\} = \{B', B\}$ and $N_1' \uparrow \{b\} = \{N_1', N_1\}$.

Formally, $A \uparrow S$ is defined inductively as follows:

1. $[\alpha] \uparrow S = \{[\alpha]\}$ for any $\alpha \in BE$.
2. Let $A = [A_1, \ldots, A_n, r]$. Define $A \uparrow S = \{ [X_1, \ldots, X_n, r] \mid \forall i : X_i \in A_i \uparrow S\} \cup \Delta$ where $\Delta = \{[hd(r)]\}$ if $hd(r) \in S$ and $\Delta = \emptyset$ otherwise.

Definition 4.6 *(***Attack Monotonicity***) We say attack relation att satisfies the axiom of* attack monotonicity *iff for each finite subset $S \subseteq BE$, for all $A, B \in AR_K$ and for each $X \in A \uparrow S$, followings hold:*

1. If $(A, B) \in att(K)$ then $(X, B) \in att(K)$.
2. If $(B, X) \in att(K)$ then $(B, A) \in att(K)$. □

Theorem 1 *Suppose attack relation* att *satisfies the axioms of attack monotonicity and context-independence. Then att also satisfies the property of irrelevance of redundant defaults.*

Proof (Sketch) Let $\omega \in BE$ and $K' = K + d$ where $d : \Rightarrow \omega$. We show that the stable belief sets of K, K' coincide.

Let E be a stable extension of K and $S = cnl(E)$. We show that S is a stable belief set of K'. Due to the context-independence, E is conflict-free wrt $att(K')$. For each argument $X \in AR_{K'}$, let $st(X)$ be the argument obtained by replacing each occurrence of defeasible rule $\Rightarrow \omega$ by ω. It is clear that $st(X) \in X \uparrow \omega$.

Let $E' = E \cup \{X \in AR_{K'}$ s.t. $st(X) \in E$ and $E \cup \{X\}$ is conflict-free wrt att(K')$\}$. It is obvious that $S = cnl(E')$. We show that E' is stable extension of K'. We show E' is conflict-free wrt att(K'). Suppose there are $X, Y \in E'$ s.t. X attacks Y wrt $att(K')$. From the attack monotonity, $st(X)$ attacks Y wrt att(K') implying that E attacks Y wrt $att(K')$. Contradiction.

Let $X \in AR_{K'} \setminus E'$. Therefore $st(X) \notin E$ or $E \cup \{X\}$ is not conflict-free wrt att(K'). If $st(X) \notin E$ then there is $A \in E$ attacks st(X) wrt att(K). Due to context independence, A attacks st(X) wrt att(K'). Due to attack monotonicity, A attacks X wrt att(K'). Hence E' attacks X wrt att(K'). Suppose now $E \cup \{X\}$ is not conflict-free wrt att(K'). If E attacks X, we are done. Suppose X attacks E wrt att(K'). Therefore st(X) attacks E wrt att(K). Hence E attacks st(X) wrt att(K). Therfore E' attacks X wrt att(K').

Let E' be a stable extension of K'. Therefore $st(E') \subseteq E'$. Hence every argument attacked by E' is attacked by st(E'). It follows immediately that st(E') is a stable extension of K wrt att(K). □.

An example of theorem 1 is example 1.2 illustrating that a violation of the property of irrelevance of redundant defaults leads to a violation of the axiom of attack monotonity.

We introduce now the cumulativity axiom. Let $\mathcal{K}$ be a class of consistent knowledge bases.

Definition 4.7 *(***Credulous Cumulativity***) We say attack relation att satisfies the axiom of* **credulous cumulativity** *for $\mathcal{K}$ iff for each $KB \in \mathcal{K}$, for each stable belief set S of KB (wrt $att(KB)$) and for each finite subset $\Omega \subseteq S$, $KB + \Omega$ belongs to $\mathcal{K}$ and S is a stable belief set of a $KB + \Omega$ (wrt $att(KB + \Omega)$).* □

There could be different versions of credulous cumulativities according to different types of belief sets. We focus in this paper on stable semantics to facilitate the comparison of argument-based and non-argument-based prioritized default reasoning.

The following lemma shows that credulous cumulativity is stronger than the consistency postulate introduced in [1].

Lemma 4.2 *Suppose attack relation att satisfies the credulous cumulativity axiom for $\mathcal{K}$. Then att also satisfies the consistency postulate for knowledge bases in $\mathcal{K}$, i.e. for each $KB \in \mathcal{K}$, the stable belief sets of KB are consistent.*

Proof Let S be a stable belief set of $KB \in \mathcal{K}$. Suppose S is inconsistent. Therefore, there is a finte subset $\Omega \subseteq S$ such that $\Omega \cup BE$ is inconsistent wrt set of strict rules of KB. Hence $KB + \Omega$ is not a consistent knowledge base, contradicting the credulous cumulativity axiom. □.

Definition 4.8 *We say attack relation att is* **well-behaved** *for a class $\mathcal{K}$ of consistent knowledge bases iff it satisfies all axioms listed in definitions 4.2, 4.3, 4.4, 4.6, 4.7 for knowledge bases from $\mathcal{K}$.*

5 An Axiomatic Analysis of Aspic$^+$

There are four attack relations in Aspic+ (also called defeats in [15, 14]) based on the notions of rebut and undercut as well as two distinct preference preorders between set of defeasible rules.

Definition 5.1 *Given two finite sets of defeasible rules Γ, Γ' and $y \in \{E, D\}$*[5]*, define: $\Gamma \trianglelefteq_y \Gamma'$ iff $\Gamma \neq \emptyset$ and one of the following conditions holds:*
1. $\Gamma' = \emptyset$
2. y = E and $\exists d \in \Gamma\; s.t. \forall d' \in \Gamma' :\; d \preceq d'$
3. y = D and $\forall d \in \Gamma\; \exists d' \in \Gamma' :\; d \preceq d'$.

We write $\Gamma \triangleleft_y \Gamma'$ iff $\Gamma \trianglelefteq_y \Gamma'$ and $\Gamma' \not\trianglelefteq_y \Gamma$. □

Definition 5.2 *Let A, B be two arguments. Let $y \in \{E, D\}$. B is* **preferred to** *A according to the* last link *(resp.* weakest link*) principle and the y-ordering, denoted by $A \sqsubseteq_{ly} B$ (resp. $A \sqsubseteq_{wy} B$) iff $ldr(A) \trianglelefteq_y ldr(B)$ (resp. $dr(A) \trianglelefteq_y dr(B)$).*

We write $A \sqsubset_{xy} B$ iff $A \sqsubseteq_{xy} B$ and $B \not\sqsubseteq_{xy} A$. □

There are accordingly four different attack relations.

Definition 5.3 $(A, B) \in att_{xy}$ *iff* 1. *A undercuts B, or*
2. *A rebuts B (at B') such that $A \not\sqsubset_{xy} B'$.*

It follows immediately

Theorem 2 *Each of the attack notions defined in definition 5.3 satisfies the axioms of subargument structure, effective rebuts and context-independence.*

Example 1.1 shows that the property of self-contradiction does not guarantee credulous cumulativity for attack relation based on weakest link and elitist-preorder. Moreover both semantics based on elitist-preorder satisfy neither the consistency nor the credulous cumulativity axioms in general as example 5.1 below shows.

Example 5.1 *Consider a knowledge base system K consisting of i) an empty base of evidence, and ii) four defeasible rules $d_i :\; \Rightarrow a_i$, $1 \leq i \leq 4$, and four strict rules $r_1 :\; a_2, a_3, a_4 \rightarrow \neg a_1,\; \dots\;,\; r_4 :\; a_1, a_2, a_3 \rightarrow \neg a_4$ and iii) $\preceq\; = \; \{d_1, d_2\} \times \{d_1, d_2\} \cup \{d_3, d_4\} \times \{d_3, d_4\}$. It is clear that $\preceq$ is a preorder and the knowledge base is consistent and closed under transposition.*

There are in total 8 arguments: $A_i = [\Rightarrow a_i]$, $1 \leq i \leq 4$ and $B_1 = [A_2, A_3, A_4 \rightarrow \neg a_1],\; \dots\;, B_4 = [A_1, A_2, A_3 \rightarrow \neg a_4]$.

It is easy to see: $\{d_1, d_3, d_4\} \triangleleft_E \{d_2\}$ and $\{d_2, d_3, d_4\} \triangleleft_E \{d_1\}$, and $\{d_1, d_2, d_3\} \triangleleft_E \{d_4\}$ and $\{d_1, d_2, d_4\} \triangleleft_E \{d_3\}$. Therefore B_i does not attack A_i for $1 \leq i \leq 4$ according to the attack relation $att_{xE}(K)$ for $x = l, w$. Therefore $att_{xE}(K) = \emptyset$. All arguments belong to the unique stable extension whose set of conclusions is $S = \{a_1, \neg a_1, \dots, a_4, \neg a_4\}$, obviously inconsistent.

Since $\Omega = \{a_1, \dots, a_4\} \subseteq S$ is inconsistent wrt set of strict rules of K, $K{+}S$ is not consistent. Hence the credulous cumulativity axiom is violated. □

Theorem 3 *Let $\mathcal{K}$ be the class of consistent knowledge bases closed under transposition or contraposition. All four attack relations in Aspic$^+$ are not well-behaved for $\mathcal{K}$. Moreover*

1. *Attack relations based on elitist preorder violate both the consistency and credulous cumulativity axioms for $\mathcal{K}$.*
2. *Attack relations based on democratic preorder violate the attack monotonicity axiom for $\mathcal{K}$.*[6]

Proof Examples 5.1, 1.2, 4.2 show the theorem for the case of closure under transposition.

To show the theorem for the case of closure under contraposition, in each example, just add for each atom α, the absurd rules $\alpha, \neg\alpha \rightarrow l$ for each literal l. The resulting knowledge bases are closed under contraposition.

In example 5.1, apart from the 8 previous arguments, it is not difficult to prove that for each new argument X, $dr(X) = ldr(X) = \{d_1, \dots, d_4\}$. Therefore $X \sqsubset_{xE} A_i, i = 1, \dots, 4, x = l, w$. Hence X does not attack any other argument. The unique stable belief set is still the same like before.

In example 4.2, adding new strict rules does not change that $N_1 \in N_1' \uparrow \{b\}$ and N_1' attacks A_2 and N_1 does not attack A_2 (wrt democratic preorder). □

We introduce in the section below a simple attack relations that is well-behaved without relying on any preference order between arguments. The new attack relation, referred to as the normal attack relation, captures the intuition of the Aspic$^+$- semantics and other non-argument-based semantics while overcoming their shortcomings.

6 Introducing the Normal Attack Relation

Definition 6.1 *Let $A, B \in AR_K$. The* normal attack relation *$att_{nr}(K) \subseteq AR_K \times AR_K$ is defined by:*

$(A, B) \in att_{nr}(K)$ iff 1. A undercuts B, or 2. A rebuts B (at B') and there is no defeasible rule $d \in ldr(A)$ such that $d \prec last(B')$.

Consider example 5.1 again. It is not difficult to see that $\prec\; = \emptyset$. Hence for each i, B_i attacks A_i and each B_j, $j \neq i$. There are four extensions $\{B_i\} \cup \{A_j \mid j \neq i\}$ for each $i = 1, .., 4$. Therefore the consistency axiom holds.

An attentive reader may ask whether in definition 6.1, d could be just any defeasible rule appearing in A. The answer is "no" as allowing d to be any defeasible rule in A would result in a violation of the credulous cumulativity axiom as illustrated in example 1.1.

[5] E, D stand for Elitist and Democratic respectively

[6] Adding for each atom α and each literal l, the absurd rules $\alpha, \neg\alpha \rightarrow l$ to the knowledge base in example 4.2 results in a knowledge base closed under contraposition.

Theorem 4 *The normal attack relation att_{nr} is well-behaved for the class of consistent knowledge bases satisfying the self-contradiction property.*

Proof(Sketch) 1. Straightforward to see that att_{nr} satisfies the axioms of subargument structure, context-independence, effective rebuts. We first prove attack monotonicity. It is not difficult to see that if A attacks B wrt att_{nr} by rebut, any argument in $A \uparrow \Omega$ also attacks B wrt att_{nr} by rebut. It should be clear that any argument C attacking an argument in $A \uparrow \Omega$ by rebut wrt att_{nr}, also rebuts A wrt att_{nr}. The axiom of credulous cumulativity (and hence consistency) follows immediately from lemma 6.1. □

The *preferences-free attack relation*, denoted by $att_f \subseteq AR_K \times AR_K$, is defined by $(A, B) \in att_f$ iff A rebuts or undercuts B.

Lemma 6.1 *Let att be an attack relation satisfying following conditions:*

1. *$att_{nr} \subseteq att \subseteq att_f$.*
2. *For all arguments A, B, B' if A attacks B (wrt att) by rebut (at B) and $last(B) = last(B')$ then A attacks B' (wrt att) by rebut (at B').*
3. *att satisfies the axioms of subargument structure and context-independence.*

Then att satisfies the credulous cumulativity axiom wrt class of consistent knowledge bases satisfying the self-contradiction property.

Proof Let E be a stable extension of (AR_K, att) and $S = cnl(E)$. We first show that S is consistent. Suppose the contrary. Since S is closed (from lemma 4.1), S is contradictory. There are two arguments $A, B \in E$ such that $cnl(B) = \neg cnl(A)$. Let $\Delta = ldr(A) \cup ldr(B)$. Therefore $hd(\Delta) \cup BE$ is inconsistent. Let Δ_0 be minimal subset of Δ such that $hd(\Delta_0) \cup BE$ is inconsistent. Since BE is consistent wrt RS, $\Delta_0 \neq \emptyset$. Let $d \in \Delta_0$ be minimal wrt $\prec$. Thus $hd(\Delta_0) \cup BE \vdash_K \neg hd(d)$ (from self-contradiction property). From lemma 4.1, $\exists C \in E$ s.t. $ldr(C) \subseteq \Delta_0$ and $cnl(C) = \neg hd(d)$. Without loss of generality, let $d \in ldr(A)$ and B be a basic defeasible subargument of A such that $last(B) = d$. It is clear that C attacks A wrt att_{nr} by rebut (at B). From $att_{nr} \subseteq att \subseteq att_f$, C attacks A wrt att by rebut (at B). Hence E attacks itself. Contradiction.

Let $\Omega \subseteq cnl(E)$. We show that $E' = E \uparrow \Omega$ is a stable extension in $K' = K + \Omega$. Since $S = cnl(E')$ is not contradictory and E is conflict-free, it is easy to see that E' is conflict-free.

Let $X \in AR_{K+\Omega} \setminus E'$. We show that E' attacks X. Without loss of generality, we could assume that all defeasible subarguments of X (except X) belong to E' and X is basic defeasible. If E' undercuts X, we are done. Suppose now E' does not undercut X. If $X \in AR_K$, we are done due to the context-independence. Suppose $X \notin AR_K$. Therefore $\exists Y \in AR_K$ s.t. $X \in Y \uparrow \Omega$ s.t. all basic defeasible subarguments of Y (except Y) belong to E. Therefore $Y \notin E$. Therefore $\exists C \in E$ s.t. $(C, Y) \in att$ by rebutting (at Y). Since $last(X) = last(Y)$, it is clear that $(C, X) \in att$. □

The following lemma reveals the relationships between the attack relations.

Lemma 6.2 *1. $att_{lE} \subseteq att_{nr} \subseteq att_{lD} \subseteq att_f$.*

2. att_{lD} satisfies the credulous cumulativity axiom wrt class of consistent knowledge bases satisfying the self-contradiction property.

3. The axiom of attack monotonicity is satisfied wrt any attack relation att_{xE}, $x \in \{l, w\}$ for any knowledge base.

4. The axioms of credulous cumulativity and attack monotonicity are independent.

Proof 1. It is obvious that $att_{lE} \subseteq att_{nr}$. We show $att_{nr} \subseteq att_{lD}$. Let C attacks B wrt att_{nr} by rebut (at B) and last(B) = d. Therefore, $\forall d' \in ldr(C) : d' \not\prec d$. We show C attacks B wrt att_{lD} by rebut (at B). If C is strict, we are done. Suppose C is defeasible and C does not attack B wrt att_{lD} by rebut (at B). Hence $ldr(C) \sqsubset_D \{d\}$. Thus $ldr(C) \sqsubseteq_D \{d\}$ and $\{d\} \not\sqsubseteq_D ldr(C)$, i.e. $\forall d' \in ldr(C) : d' \preceq d$ and $\forall d' \in ldr(C) : d \not\preceq d'$. Hence $\forall d' \in ldr(C) : d' \prec d$. Contradiction.

2. The assertion follows immediately from the first and lemma 6.1.

3. It is not difficult to see that for $X' \subseteq X$, if $X' \lhd_E Y$ then $X \lhd_E Y$, and if $Y \lhd_E X$ then $Y \lhd_E X'$.

Let $S \subseteq BE$ and $A \in AR_K$ and $A' \in A \uparrow S$. Let $f_w = dr$ and $f_l = ldr$. Suppose $(A, B) \in att_{xE}$. If A undercuts B then A' also undercuts B. Hence $(A', B) \in att_{xE}$. Let A rebuts B (at B').

Therefore $f_x(A) \not\lhd_E f_x(B')$, From $f_x(A') \subseteq f_x(A)$ it follows $f_x(A') \not\lhd_E f_x(B')$. Hence $(A', B) \in att_{xE}$.

Suppose $(C, A') \in att_{xE}$. If C undercuts A' then C undercuts A. Let C rebuts A' (at B'). Therefore $C \not\sqsubset_{xE} B'$, i.e. $f_x(C) \not\lhd_E f_x(B')$. From $A' \in A \uparrow S$, there is a basic defeasible subargument B of A such that $B' \in B \uparrow S$. It follows $f_x(C) \not\lhd_E f_x(B)$. Hence $(C, B) \in att_{xE}$. Therefore $(C, A) \in att_{xE}$.

4. It follows immediately from the previous two assertions in this lemma and examples 1.2, 5.1. □

7 Operational Interpretation of Rules Ordering

Preference orders between rules in prioritized default logics or logic programming are viewed in [4, 18] as specifying application orders of rules. This operational reading of preferences is sound wrt normal semantics. It is also complete for the class of well-ranked knowledge bases. We first adapt the definitions in [4, 18] to structured argumentation below.

Definition 7.1 *A stable extension E of (AR_K, att_f) is said to be an* **enumeration-based extension** *of K if there is an enumeration $(d_i)_{i \geq 1}$ of $\Gamma_E = \{d \in RD \mid$ d appears in some argument of E $\}$ such that for all i,j, we have:*

1. *$\{hd(d_k) | k < i\} \cup BE \vdash_K bd(d_i)$;*
2. *if $d_i \prec d_j$ then $j < i$;*
3. *if $d_i \prec d$ and $d \in RD \setminus \Gamma_E$ then $bd(d) \not\subseteq cnl(E)$ or $\{hd(d_k) | k < i\} \cup BE \vdash_K \neg hd(d)$ or $\{hd(d_k) | k < i\} \cup BE \vdash_K ab_d$*

The following theorem shows the soundness of enumeration-based semantics wrt normal semantics.

Theorem 5 *Every enumeration-based extension of K is a stable extension of K wrt attack relation att_{nr}.*

Proof (Sketch) Let E be an enumeration-based extension of K. We show that E is also a stable extension wrt (AR_K, att_{nr}). As E is a stable extension of $AF_f = (AR_K, att_f)$, E is conflict-free wrt att_{nr}. As att_f satisfies the axiom of subargument structure, cnl(E) is closed.

Let A be an argument not belonging to E. We show E attacks A wrt att_{nr}. Since E is a stable extension of $AF_f = (AR_K, att_f)$, there is an argument $B \in E$ that either undercuts or rebuts A. If

B undercuts A then it is obvious B attacks A wrt att_{nr}. Suppose B rebuts A. Without loss of generality, we can assume that A is basic defeasible and all subarguments of A except A itself belong to E. Let $d = last(A)$. Therefore $bd(d) \subseteq cnl(E)$. If B also attacks A wrt att_{nr} then we are done. Suppose now that B does not attack A wrt att_{nr}. Therefore there is $d' \in ldr(B)$ such that $d' \prec d$. Let $(d_i)_{i\geq 1}$ be an enumeration of Γ_E as described in definition 7.1. It is clear that $d \notin \Gamma_E$ and $d' \in \Gamma_E$. Hence $\exists i$ s.t. $d' = d_i$. Let $n = min\{ j \mid d_j \prec d\}$. Hence $d_n \prec d$. From definition 7.1, it follows $\{hd(d_k) | k < n\} \cup BE \vdash_K \neg hd(d)$. Therefore, from the closure of E, there is an argument $C \in E$ such that $dr(C) \subseteq \{hd(d_k) | k < n\}$ and $cnl(C) = \neg hd(d)$. From the definition of n, it follows that there is no rule in $ldr(C)$ that is strictly less preferred than d. Therefore C attacks A wrt att_{nr}. □

When the operational interpretation of rules preferences interferes with the basic control mechanism of "applying a rule when its premises are derived", there could be no enumeration-based extension. For illustration, consider a knowledge base consisting of just two defeasible rules $d :\Rightarrow a$, $d' :\ a \Rightarrow b$ with $d \prec d'$. There is no enumeration-based extension in this case.

When the basic control mechanism and the operational reading of the rule preferences do not interfere, the enumeration-based semantics coincide with the normal semantics.

A preference order $\preceq$ is said to be **ranked** iff there is a ranking function ρ assigning non-negative integers to defeasible rules in RD such that for all $d, d' \in RD$, $d \preceq d'$ iff $\rho(d') \leq \rho(d)$.

A knowledge base K is said to be **well-ranked** iff its preference preorder $\preceq$ is ranked with a ranking function ρ such that following conditions are satisfied:

1. for each basic defeasible argument A, for each defeasible rule d occurring in A and different to $last(A)$, $\rho(d) < \rho(last(A))$.

2. For each argument A such that $cnl(A) = ab_d$ for some defeasible rule d, $\rho(A) \leq \rho(d)$ where $\rho(A)$ is the maximum of the ranks of the defeasible rules appearing in A. □

Theorem 6 *Suppose the preference preorder $\preceq$ of K is well-ranked and K is a consistent knowledge base satisfying the self-contradiction property. Then each stable extension (wrt att_{nr}) is an enumeration-based extension.*

Proof (Sketch) Let E be a stable extension wrt att_{nr}. From lemmas 6.1, 4.2, $cnl(E)$ is consistent. Hence E is a stable extension of (AR_K, att_f). Let $\Gamma_i = \{d \in \Gamma_E \mid \rho(d) = i\}$. Define an enumeration of Γ_E as follows:

1. List all rules in Γ_0 resulting in $(d_i)_{i\leq n_0}$.

2. Suppose the list $(d_i)_{i\leq n_i}$ of rules in $\Gamma_0 \cup \ldots \cup \Gamma_i$, $0 \leq i$, has been constructed. $(d_i)_{i\leq n_{i+1}}$ is obtained by from $(d_i)_{i\leq n_i}$ by appending to it a list of rules in Γ_{i+1}.

It is obvious that if $d_i \prec d_j$ then $j < i$; It is not difficult to prove $\{hd(d_k) | k < i\} \cup BE \vdash_K bd(d_i)$ by induction on i using the following property.

Property Let A be a basic defeasible argument and $d = last(A)$. Then $hd(dr(A) \setminus \{d\}) \cup BE \vdash_K bd(d)$.

Let $d \in RD \setminus \Gamma_E$ such that $bd(d) \subseteq cnl(E)$ and there is j such that $\rho(d) < \rho(d_j)$. Let B be a basic defeasible argument whose last rule is d and whose proper subarguments all belong to E. Since E is stable, there is an argument $A \in E$ attacking B wrt att_{nr}. From $\rho(A) \leq \rho(d)$, it follows immediately that all defeasble rules in A are listed before d_j. Therefore $\{hd(d_k) | k < j\} \cup BE \vdash_K \neg hd(d)$ or $\{hd(d_k) | k < j\} \cup BE \vdash_K ab_d$. Hence $(d_i)_{i\geq 1}$ is an enumeration of Γ_E as defined in definition 7.1. □

8 Discussions

Different kinds of rebuts and attacks have been proposed for structured argumentation with defeasible rules [1] or with classical logics [12] or defeasible logic programming [9]. It would be interesting to investigate how our results and the systems in [1, 12] could be combined.

Two well-known principles specifying properties sensible semantics of prioritized default reasoning should satisfy have been studied in [7]. The two principles could be viewed as the "equivalents" of the effective rebut and context-independence axioms in the context of extended logic programs with answer set semantics. A formal elaboration of these connections will be given a the full version of the paper. Other semantics for prioritized default reasoning have also been proposed in [17, 10]. It would be interesting to work out the connection with our approach. Initial results show that these approaches are not fully compatible with the context-independence axiom.

REFERENCES

[1] L. Amgoud and M. Caminada, 'On the evaluation of argumentation formalisms', *Artificial Intelligence*, **171**, 286–310, (2007).
[2] G. Brewka, 'Preferred subtheories: An extended logical framework for default reasoning', in *IJCAI*, pp. 1043–1048, (1989).
[3] G. Brewka, I. Niemelä, and M. Truszczynski, 'Preferences and nonmonotonic reasoning', *AI Magazine*, **29**(4), 69–78, (2008).
[4] J.P. Delgrande, T. Schaub, and H. Tompits, 'A framework for compiling preferences in logic programs', *Theory and Practice of Logic Programming*, 129–187, (2003).
[5] P.M. Dung, 'On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person gamesceptability of arguments and its fundamental role in nonmono- tonic reasoning, logic programming and n-person games', *Artif. Intell.*, **77**(2), 321–358, (1995).
[6] P.M. Dung and P.M. Thang, 'Closure and consistency in logic-associated argumentation', *Journal of Artificial Intelligence Research*, **49**(79-109), (2014).
[7] T. Eiter and G. Brewka, 'Preferred answer sets for extended logic programs', *Artificial Intelligence*, **109**, 297–356, (1999).
[8] D. M. Gabbay, 'Theoretical foundations for nonmonotonic reasoning in expert systems', in *Logic and Models of Concurrent Systems*, ed., K.R. Apt. Springer Verlag, (1985).
[9] A.J. Garcia and G.R. Simari, 'Defeasible logic programming: An argumentative approach', *TPLP*, **4**(1-2), 95–138, (2004).
[10] H. Geffner and J. Pearl, 'Conditional entailment: Bridging two approaches to default reasoning', *Artificial Intelligence*, **53**, 209–244, (1992).
[11] M. Gelfond and T.C. Son, 'Reasoning with prioritized defaults', in *LPKR*, pp. 164–223, (1997).
[12] N. Gorogiannis and A. Hunter, 'Instantiating abstract argumentation with classical logic arguments: Postulates and properties', *Artificial Intelligence*, **175**(9-10), 1479–1497, (2011).
[13] S. Kraus, D. Lehmann, and M. Magidor, 'Nonmonotonic reasoning, preferential models and cumulative logics', *Artificial Intelligence*, **44**, 167–207, (1990).
[14] S. Modgil and H. Prakken, 'A general account of argumentation with preferences', *Artificial Intelligence*, (2012).
[15] S.J. Modgil and H. Prakken, 'The aspic+ framework for structured argumentation: a tutorial', *Argument and Computation*, **5**, 31–62, (2014).
[16] J.L. Pollock, 'Defeasible reasoning', *Cognitive Science*, **11**(4), 481–518, (1987).
[17] J. Rintanen, 'Lexicographic priorities in default logic', *Artificial Intelligence*, **102**(2), 221–265, (1998).
[18] T. Schaub and K. Wang, 'A comparative study of logic programs with preference: Preliminary report', in *Answer Set Programming*, eds., A. Provetti and T.C. Son, (2001).