A Canonical Semantics for Structured Argumentation with Priorities

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Abstract. Due to a proliferation and diversity of approaches to reasoning with prioritized rules, ordinary properties have been introduced recently for characterization and evaluation of the proposed semantics. While ordinary properties are helpful, a fundamental question of whether they are sufficient to identify a common semantics underlining reasoning with priorities remains open. In this paper we address this question by introducing a new simple and intuitive property of inconsistency-resolving and slightly adapting other ordinary properties to show that they together indeed determine an unique canonical attack relation that could be viewed as defining an uniquely defined common semantics for reasoning with prioritized rules.

Keywords. regular properties, attack relation assignment, semilattice

1. Introduction

Reasoning with prioritized rules is an important and prevalent paradigm in practical reasoning like legal reasoning or commonsense reasoning [3,5,11,14]. Due to a proliferation and diversity of approaches [3,6,5,20,11,24,14,22,23], it is important to establish general principles for characterizing and evaluation of the proposed semantics. Earlier, Brewka and Eiter [6] have proposed two principles for non-argument-based approaches. Caminada and Amgoud [8] have introduced the postulates of consistency and closure that the extensions of argument-based systems should satisfy. A subargument closure postulate stating that any extension should contain all subarguments of its arguments has been studied in [22,21,1]. Though the three proposed postulates are helpful, they are not sufficient to guarantee intuitive semantics as they do not take into account the preferences of defeasible rules. To address this problem, Dung [17,12] has proposed a set of simple properties, referred to as ordinary properties in [12] and argued that they capture the natural intuitions of reasoning with prioritized rules. Still, a fundamental question of whether the proposed properties are sufficient to identify a common semantics underlining reasoning with priorities remains open. In this paper we address this question by introducing a new simple and intuitive property of inconsistency-resolving and showing that this property together with some other ordinary properties indeed determine an unique canonical attack relation that could be viewed as defining a common unique semantics for reasoning with prioritized rules.

The paper is organized as follows. We recall in the next section the key concepts and notions on which the paper is based. We then introduce the important property of inconsistency-resolving in the following section. In section 4, we introduce the new and novel concepts of regular attack relations and regular attack relation assignments. In section 5, we study the semilattice of regular attack relation assignments and propose the canonical semantics. We then conclude.

2. Preliminaries

2.1 Abstract Argumentation and Semilattice

An abstract argumentation framework [16] is defined simply as a pair (AR, att) where AR is a set of arguments and $att \subseteq AR \times AR$ where $(A, B) \in att$ means that A *attacks* B. A set of argument S attacks (or is attacked by) an argument A (or a set of arguments R) if some argument in S attacks (or is attacked by) A (or some argument in R); S is *conflict-free* if it does not attack itself. A set of arguments S *defends* an argument A if S attacks each attack against A. S is *admissible* if S is conflict-free and defends each argument in it. A *complete extension* is an admissible set of arguments containing each argument it defends. A *stable extension* is a conflict-free set of arguments that attacks every argument not belonging to it.

A partial order (i.e. a reflexive, transitive and antisymmetric relation) $\leq$ on a set S is a **upper-semilattice** (resp. **lower-semilattice**) [10] iff each subset X of S has a supremum denoted by $\sqcup X$ (resp. infimum denoted by $\sqcap X$) wrt $\leq$. The upper (resp. lower) semilattice is often denoted as a triple $(S, \leq, \sqcup)$ (resp. $(S, \leq, \sqcap)$). It follows immediately that each upper (resp. lower) semilattice S has an unique greatest (resp. least) element denoted by $\sqcup S$ (resp. $\sqcap S$).

2.2 Defeasible Knowledge Bases

In this section and the following one, we recall the basic notions and notations on knowledge bases from [12,22]. We assume a non-empty set $\mathcal{L}$ of ground atoms (also called a positive literal) and their classical negations (also called negative literals). A set of literals is said to be *contradictory* iff it contains an atom a and its negation $\neg a$. We distinguish between *domain atoms* representing propositions about the concerned domains and *non-domain atoms* of the form ab_d representing the non-applicability of defeasible rule d (even if the premises of d hold).

Following [22,23,19,20,25,12], we distinguish between strict and defeasible rules. A *defeasible* (resp. *strict*) rule r is of the form $b_1, \ldots, b_n \Rightarrow h$ (resp. $b_1, \ldots, b_n \rightarrow h$) where $b_1, \ldots, b_n$ are domain literals and h is a domain literal or an atom of the form ab_d. The set $\{b_1, \ldots, b_n\}$ (resp. the literal h) is referred to as the *body* (resp. *head*) of r and denoted by bd(r) (resp. hd(r)).

Definition 1 (1.) *A* **rule-based system** *is defined as a triple $\mathcal{R} = (RS, RD, \preceq)$ where* 1) *RS is a set of strict rules,* 2) *RD is a set of defeasible rules, and* 3) *$\preceq$ is a transitive relation over RD representing the preferences between defeasible rules, whose strict core is $\prec$ (i.e. $d \prec d'$ iff $d \preceq d'$ and $d' \npreceq d$ for $d, d' \in RD$.)*
(2.) *A* **knowledge base** *is defined as a pair $K = (\mathcal{R}, BE)$ consisting of a rule-based system $\mathcal{R}$, and a set of ground domain literals BE, the* base of evidence *of K, representing unchallenged observations, facts ect..*

For convenience, knowledge base K is often written directly as a quadruple $(RS, RD, \preceq, BE)$ where RS, RD, $\preceq$ or BE of K are often referred to by $RS_K, RD_K, \preceq_K$ or BE_K respectively.
(3.) *A knowledge base K is* **basic** *if its precedence relation is empty (i.e. $\preceq_K = \emptyset$).*

Definition 2 *Let $K = (RS, RD, \preceq, BE)$ be a knowledge base. An* **argument** *wrt K is a proof tree defined inductively as follows:*
(1.) *For each $\alpha \in BE$, $[\alpha]$ is an argument with conclusion α.*
(2.) *Let r be a rule of the forms $\alpha_1, \ldots, \alpha_n \rightarrow / \Rightarrow \alpha$, $n \geq 0$, from $RS \cup RD$ and $A_1, \ldots, A_n$ be arguments with conclusions α_i, $1 \leq i \leq n$, respectively. Then $A = [A_1, \ldots, A_n, r]$ is an argument with* **conclusion** *α and* **last rule** *r denoted by* **cnl(A)** *and* **last(A)** *respectively.*
(3.) *Each argument wrt K is obtained by applying the above steps 1, 2 finitely many times.*

Example 1 *Consider a rule-based system $\mathcal{R}$ (adapted from [6,7,12]) whose sets of rules consisting of three defeasible rules $d_1 : Dean \Rightarrow Professor$, $d_2 : Professor \Rightarrow Teach$, $d_3 : Administrator \Rightarrow \neg Teach$ and two strict rules $r : Dean \rightarrow Administrator$, $r' : \neg Administrator \rightarrow \neg Dean$ together with a precedence relation consisting of just $d_2 \prec d_3$. Suppose we know some dean who is also a professor. The considered knowledge base is represented by $K = (RS, RD, \preceq, BE)$ with $RS = \{r, r'\}$, $RD = \{d_1, d_2, d_3\}$, $\preceq = \{(d_2, d_3)\}$ and $BE = \{D, P\}$ (D,P,T,A stand for Dean, Professor , Teach and Administrator respectively). Relevant arguments can be found in figure 1 where $A_1 = [[D], d_1]$, $A_2 = [A_1, d_2]$, $A'_2 = [[P], d_2]$, $A_3 = [\,[[D], r], d_3]$.*

$$\begin{array}{cccc} A_1 : P & A_2 : T & A_3 : \neg T & A'_2 : T \\ \uparrow d_1 & \uparrow d_2 & \uparrow d_3 & \uparrow d_2 \\ D & P & A & P \\ & \uparrow d_1 & \uparrow r & \\ & D & D & \end{array}$$

Figure 1. Dean Example

Notation 1 *The set of all arguments wrt a knowledge base K is denoted by* $\mathbf{AR_K}$. *The set of the conclusions of arguments in a set $S \subseteq AR_K$ is denoted by* **cnl(S)**.

A **strict argument** *is an argument containing no defeasible rule. An argument is* **defeasible** *iff it is not strict. A defeasible argument A is called* **basic defeasible** *iff $last(A)$ is defeasible. For any argument A, the set of defeasible rules appearing in A is denoted by* **dr(A)**. *The set of last defeasible rules in A, denoted by* **ldr(A)**, *is $\{last(A)\}$ if A is basic defeasible, otherwise it is equal $ldr(A_1) \cup \ldots \cup ldr(A_n)$ where $A = [A_1, \ldots, A_n, r]$. An argument B is a* **subargument** *of an argument A iff $B = A$ or $A = [A_1, \ldots, A_n, r]$ and B is a subargument of some A_i. B is a* proper subargument *of A if B is a subargument of A and $B \neq A$.*

Definition 3 (1.) *The* **closure** *of a set of literals $X \subseteq \mathcal{L}$ wrt knowledge base K, denoted by $CN_K(X)$, is the union of X and the set of conclusions of all strict arguments wrt knowledge base $(RS_K, RD_K, \preceq_K, X_{dom})$ with X_{dom} (the set of all domain literals in X) acting as a base of evidence. X is said to be* **closed** *iff $X = CN_K(X)$. X is said to be* **inconsistent** *iff its closure $CN_K(X)$ is contradictory. X is* **consistent** *iff it is not inconsistent. We also often write $X \vdash_K l$ iff $l \in CN_K(X)$.*
(2.) *K is said to be* **consistent** *iff its base of evidence BE_K is consistent.*

As the notions of closure, consistency depend only on the set of strict rules in the knowledge base, we often write $X \vdash_{RS} l$ or $l \in CN_{RS}(X)$ for $X \vdash_K l$ or $l \in CN_K(X)$ respectively.

Definition 4 *Let $\mathcal{R} = (RS, RD, \preceq)$ be a rule-based system and $K = (\mathcal{R}, BE)$ be a knowledge base.*
(1.) *$\mathcal{R}$ and K are said to be* closed under transposition *[8] iff for each strict rule of the form $b_1, \ldots, b_n \to h$ in RS s.t. h is a domain literal, all the rules of the forms $b_1, \ldots, b_{i-1}, \neg h, b_{i+1}, \ldots, b_n \to \neg b_i$, $1 \leq i \leq n$, also belong to RS.*
(2.) *$\mathcal{R}$ and K are said to be* closed under contraposition *[24,23] iff for each set of domain literals S, each domain literal λ, if $S \vdash_{RS} \lambda$ then for each $\sigma \in S$, $S \setminus \{\sigma\} \cup \{\neg\lambda\} \vdash_{RS} \neg\sigma$.*
(3.) *$\mathcal{R}$ and K are said to satisfy the* self-contradiction property *[15] iff for each minimal inconsistent set of domain literals $X \subseteq \mathcal{L}$, for each $x \in X$, it holds: $X \vdash_{RS} \neg x$.*

Lemma 1 *([12]) Let $\mathcal{R}$ be a rule-based system that is closed under transposition or contraposition. Then $\mathcal{R}$ satisfies the property of self-contradiction.*

Definition 5 *(**Attack Relation**) An* attack relation *for a knowledge base K is a relation $att \subseteq AR_K \times AR_K$ such that there is no attack against strict arguments, i.e. for each strict argument $B \in AR_K$, there is no argument $A \in AR_K$ such that $(A, B) \in att$.*
For convenience, we often say A attacks B wrt att *for $(A, B) \in att$.*

2.3 Basic Postulates

We recall the postulates of consistency, closure and subargument closure from [8,22,1,21] where we combine the postulate of closure [8] and the postulate of subargument closure [22,1,21] into one.

Definition 6 *Let att be an attack relation for a knowledge base K.*

- *att is said to satisfy the* **consistency postulate** *iff for each complete extension E of (AR_K, att), the set $cnl(E)$ of conclusions of arguments in E is consistent.*
- *att is said to satisfy the* **closure postulate** *iff for each complete extension E of (AR_K, att), the set $cnl(E)$ of conclusions of arguments in E is closed and E contains all subarguments of its arguments.*

For ease of reference, the above two postulates are often referred to as **basic postulates**.

3. Sufficient Properties for Basic Postulates

As the basic postulates are more about the "output" of attack relations rather than about their structure, we present below two simple properties about the structure of attack relation that ensures the holding of the basic postulates. We first introduce some simple notations.

We say A *undercuts* B (at B') iff B' is basic defeasible and $cnl(A) = ab_{last(B')}$. We also say A **rebuts** B (at B') iff B' is a basic defeasible subargument of B and the conclusions of A and B' are contradictory [8,22].

An argument A is said to be **generated by** a set S of arguments iff all basic defeasible subarguments of A are subarguments of arguments in S. For an example, let $S = \{B_0, B_1\}$ (see figure 2). Let consider A_0. The set of basic defeasible subarguments of A_0 is $\{\, [d_0]\}$. It is clear that $[d_0]$ is a subargument of B_0. Hence A_0 is generated by S. Similarly, A_1 is also generated by S.

We say A *directly attacks* B if A attacks B and A does not attack any proper subargument of B.

Definition 7 *(***Strong Subargument Structure***) Attack relation att is said to satisfy the* property of strong subargument structure *for K iff for all $A, B \in AR_K$, followings hold:*
(1.) *If A undercuts B then A attacks B wrt att.*
(2.) *A attacks B (wrt att) iff A attacks a basic defeasible subargument of B (wrt att).*
(3.) *If A directly attacks B (wrt att) then A undercuts B (at B) or rebuts B (at B).*

We present the first result showing that strong subargument property is sufficient to guarantee the postulate of closure.

Lemma 2 *Let att be an attack relation for knowledge base K satisfying the property of strong subargument structure. Then att satisfies the postulate of closure.*

Proof (Sketch) From condition 2 in definition 7, it follows that each attack against an argument generated by complete extension E is an attack against E. The lemma holds obviously. □

A set S of arguments is said to be *inconsistent* if the set of the conclusions of its arguments, $cnl(S)$, is inconsistent. We introduce below a new simple property of inconsistency resolving, a key result of the paper.

Definition 8 *(***Inconsistency Resolving***) We say attack relation assignment att satisfies the* inconsistency-resolving property *for K iff for each finite set of arguments $S \subseteq AR_K$, if S is inconsistent then S is attacked (wrt $att(K)$) by some argument generated by S.*

As we will show later, the inconsistency-resolving property is satisfied by common conditions like closure under transposition, or contradiction or the property of self-contradiction.

Example 2 *Consider the basic knowledge base K consisting of just the rules appearing in arguments in figure 2. The set $S = \{B_0, B_1\}$ is inconsistent. The argument A_0 is generated by S. Let $att = \{(X, Y) \mid X \text{ rebuts } Y\}$. It is obvious that S is attacked by A_0. It is clear that att is inconsistency-resolving.*

We present now the first important result of this paper.

Theorem 1 *Let att, att' be attack relations for knowledge base K.* (1.) *If $att \subseteq att'$ and att is inconsistency-resolving for K then att' is also inconsistency-resolving for K;*
(2.) *If att satisfies the strong subargument structure and inconsistency-resolving then att satisfies the postulate of consistency.*

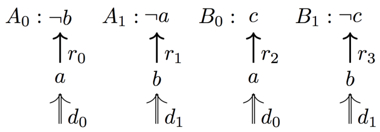

Figure 2. Generated Arguments

Proof (Sketch) Assertion 1 follows easily from the definition of inconsistency-resolving. We only need to show assertion 2. From condition 2 in definition 7, it follows that each argument generated by a complete extension E belongs to E. Therefore, if E is inconsistent then E is conflicting. Since E is not conflicting, E is hence consistent. □

4. Regular Attack Relation Assignments

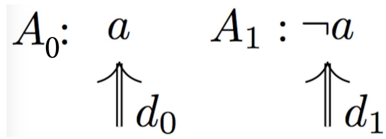

Figure 3. Effective Rebuts

Dung [17,12] has proposed the ordinary properties to capture the intuition of prioritized rules. We recall and adapt them below. We also motivate and explain their intuitions. We then define two new novel concepts of regular attack relations and regular attack relation assignments that lie at the heart of the semantics of prioritized rules.

4.1 A Minimal Interpretation of Priorities

We first recall from [12] the effective rebut property stating a "minimal interpretation" of a preference $d_0 \prec d_1$ that in situations when both are applicable but accepting both d_0, d_1 is not possible, d_1 should be preferred. In figure 3, the effective rebut property dictates that A_1 attacks A_0 but not vice versa.

Definition 9 *(***Effective Rebut***) We say that attack relation att satisfies the* effective rebut property *for a knowledge base K iff for all arguments $A_0, A_1 \in AR_K$ such that each A_i, $i = 0, 1$, contains exactly one defeasible rule d_i (i.e. $dr(A_i) = \{d_i\}$), and A_0 rebuts A_1, it holds that A_0 attacks A_1 wrt att iff $d_0 \not\prec d_1$.*

4.2 Propagating Attacks

Example 3 *Consider the knowledge base in example 1. While the effective rebut property determines that A_3 attacks A'_2 (see figure 1) but not vice versa (because $d_2 \prec d_3$), it does not say whether A_3 attack A_2.*

Looking at the structure of A_2, A'_2, we can say that A_2 is a weakening of A'_2 as the undisputed fact P on which A'_2 is based is replaced by the defeasible belief P (supported by argument A_1). Therefore if A_3 attacks A'_2 then it is natural to expect that A_3 should attack A_2 too.

The above analysis also shows that attacks generated by the effective rebut property, could be propagated to other arguments based on a notion of weakening of arguments. We recall this notion as well as the associated property of attack monotonicity from [12] below.

Let $A, B \in AR_K$ and $AS \subseteq AR_K$. Intuitively, B is a weakening of A by AS if B is obtained by replacing zero, one or more premises of A by arguments in AS whose conclusions coincide with the premises.

Definition 10 *B is said to be* **a weakening of** *A by AS iff*

(1.) *$A = [\alpha]$ for $\alpha \in BE$, and ($B = [\alpha]$ or $B \in AS$ with $cnl(B) = \alpha$), or*

(2.) *$A = [A_1, \ldots, A_n, r]$ and $B = [B_1, \ldots, B_n, r]$ where each B_i is a weakening of A_i by AS.*

By $A \downarrow AS$ we denote the set of all weakenings of A by AS.

For an illustration, consider again the arguments in figure 1. It is clear that $[P] \downarrow \{A_1\} = \{[P], A_1\}$, $A'_2 \downarrow \{A_1\} = \{A'_2, A_2\}$.

The attack monotonicity property states that if an argument A attacks an argument B then A also attacks all weakening of B. Moreover if a weakening of A attacks B then A also attacks B.

Definition 11 *(***Attack Monotonicity***) We say attack relation* att *satisfies the* property of attack monotonicity *for knowledge base K iff for all $A, B \in AR_K$ and for each weakening C of A, for each weakening D of B, the following assertions hold:*

1. *If $(A, B) \in att$ then $(A, D) \in att$.*
2. *If $(C, B) \in att$ then $(A, B) \in att$.*

We next recall the link-oriented property in [12] which is based on an intuition that attacks are directed towards links in arguments implying that if an argument A attacks an argument B then it should attack some part of B.

Definition 12 *(***Link-Orientation***) We say that attack relation att satisfies the* property of link-orientation *for K iff for all arguments $A, B, C \in AR_K$ such that C is a weakening of B by $AS \subseteq AR_K$ (i.e. $C \in B \downarrow AS$), it holds that if A attacks C (wrt att) and A does not attack AS (wrt att) then A attacks B (wrt att).*

In real world conversation, if you claim that my argument is wrong, I would naturally ask which part of my argument is wrong. The link-oriented property could be viewed as representing this intuition.

Example 4 *Consider again arguments in figure 1. Suppose d_2 is now preferred to d_3 (i.e. $d_3 \prec d_2$). The effective rebut property dictates that A_3 does not attack A'_2. Does A_3 still attack A_2 ? Suppose A_3 attacks A_2. Since A_3 does not attack A_1 that is a subargument of A_2, we expect that A_3 should attack some other part of A_2. In other words, we expect that A_3 attacks A'_2. But this is a contradiction to the effective rebut property stating that A'_2 attack A_3 but not vice versa. Hence A'_3 does not attack A_2.*

In other words, the link-orientation property has propagated the "non-attack relation" between A_3, A'_2 to a "non-attack relation" between A_3, A_2.

We present below a new and novel concept of regular attack relations.

Definition 13 *An attack relation is said to be* **regular** *if it satisfies to the properties of inconsistency-resolving and strong subargument structure together with the properties of effective rebuts, attack monotonicity and link-orientation.*

4.3 Attack Relation Assignments: Propagating Attacks Across Knowledge Bases

While regular attack relations are natural and intuitive, they are still not sufficient for determining an intuitive semantics of prioritized rules. The example below illustrates this point.

Example 5 *Consider a knowledge base K_0 obtained from knowledge base K in example 1 by revising the evidence base to $BE = \{D\}$. It is clear that arguments A_1, A_2, A_3 belong to AR_{K_0} while A'_2 is not an argument in AR_{K_0}.*

As A'_2 does not belong to AR_{K_0}, the effective rebuts property does not "generate" any attacks between arguments in AR_{K_0}. How could we determine the attack relation for K_0.

As both A_2, A_3 belong to AR_K, AR_{K_0} and the two knowledge bases K_0, K have identical rule-based system, we expect that the attack relations between their common arguments should be identical. In other words, because A_3 attacks A_2 wrt K (see example 3), A_3 should attack A_2 also wrt K_0. This intuition is captured by the context-independence property in [12] linking attack relations between arguments across the boundary of knowledge bases.

The example also indicates that attack relations of knowledge bases with the same rule-based system should be considered together. This motivates the introduction of the attack relation assignment in definitions 14,15.

Definition 14 *Let $\mathcal{R} = (RS, RD, \preceq)$ be a rule-based system. The class consisting of all consistent knowledge bases of the form $(\mathcal{R}, BE)$ is denoted by $\mathcal{C}_{\mathcal{R}}$.*

A rule-based system $\mathcal{R}$ is said to be **sensible** iff the set $\mathcal{C}_{\mathcal{R}}$ is not empty. *From now on, whenever we mention a rule-based system, we mean a sensible one.*

Definition 15 *(***Attack Relation Assignment***) An* attack relation assignment atts *for a rule-based system $\mathcal{R}$ is a function assigning to each knowledge base $K \in \mathcal{C}_{\mathcal{R}}$ an attack relation $atts(K) \subseteq AR_K \times AR_K$.*

We next recall the context-independence property stating that the attack relation between two arguments depends only on the rules appearing in them and their preferences.

Definition 16 *(***Context-Independence***) We say attack relation assignment* atts *for a rule-based system $\mathcal{R}$ satisfies the property of* context-independence *iff for any two knowledge bases $K, K' \in \mathcal{C}_{\mathcal{R}}$ and for any two arguments A, B from $AR_K \cap AR_{K'}$, it holds that $(A, B) \in atts(K)$ iff $(A, B) \in atts(K')$*

The context-independence property is commonly accepted in many well-known argument-based systems like the assumption-based framework [4,18], the ASPIC+ approach [24,22].

We can now present a central contribution of this paper, the introduction of the regular attack relation assignments.

Definition 17 *(***Regular Attack Relation Assignments***) An attack relation assignment atts for a rule-based system $\mathcal{R}$ is said to be* **regular** *iff it satisfies the property of context-independence and for each knowledge base $K \in \mathcal{C}_{\mathcal{R}}$, $atts(K)$ is regular.*

The set of all regular attack relation assignments for $\mathcal{R}$ is denoted by $RAA_{\mathcal{R}}$.

For attack relation assignments $atts, atts'$, define $atts \subseteq atts'$ iff $\forall K \in \mathcal{C}_{\mathcal{R}}$, $atts(K) \subseteq atts'(K)$.

Minimal Removal Intuition

A key purpose of introducing priorities between defeasible rules is to remove certain undesired attacks while keeping the set of removed attacks to a minimum. The following very simple example illustrates the idea.

$$\begin{array}{cccc} A: a & A_1: \neg a & B: b & B_1: \neg b \\ \Uparrow_{d_0} & \Uparrow_{d_1} & \Uparrow_{d_2} & \Uparrow_{d_3} \end{array}$$

Figure 4. Minimal Removal

Example 6 *Consider a knowledge base consisting of just four defeasible rules and four arguments A, A_1, B, B_1 as seen in figure 4. Without any preference between the rules, we have A, A_1 attack each other. Similarly B, B_1 attack each other.*

Suppose that for whatever reason d_3 is strictly less preferred than d_2 (i.e. $d_3 \prec d_2$). The introduction of the preference $d_3 \prec d_2$ in essence means that the attack of B_1 against B should be removed, but it does not say anything about the other attacks. Hence they should be kept, i.e. the attacks that should be removed should be kept to a minimum.

Let $\mathcal{R}$ be a rule-based system and $K \in \mathcal{C}_{\mathcal{R}}$. The *basic attack relation assignment* for $\mathcal{R}$, denoted by $Batts$ is defined by: $\forall K \in \mathcal{C}_{\mathcal{R}}$, $Batts(K) = \{(A, B) \mid A$ undercuts or rebuts B$\}$. Further let atts be a regular attack relation assignment. From the strong subargument structure property, it is clear that $atts \subseteq Batts$. $\forall K \in \mathcal{C}_{\mathcal{R}}$, the set $Batts(K) \setminus atts(K)$ could be viewed as the set of attacks removed from $Batts(K)$ due to the priorities between defeasible rules.

Combining the "minimal-removal intuition" with the concept of regular attack relation assignment suggests that the semantics of $\mathcal{R}$ should be captured by regular attack relations atts such that $\forall K \in \mathcal{C}_{\mathcal{R}}$, the set $Batts(K) \setminus atts(K)$ is minimal, or equivalently the set $atts(K)$ is maximal. As we will see in the next section, such maximal attack relation assignment indeed exists.

5. The Upper Semilattice of Regular Attack Relation Assignments

From now on until the end of this section, we assume an arbitrary but fixed rule-based system $\mathcal{R} = (RS, RD, \preceq)$.

Let $\mathcal{A}$ be a non-empty set of attack relation assignments for $RAA_{\mathcal{R}}$. Define $\sqcup\mathcal{A}$ by: $\forall K \in \mathcal{C}_{\mathcal{R}}$: $\quad (\sqcup\mathcal{A})(K) = \bigcup\{\, atts(K) \mid atts \in \mathcal{A} \,\}$

The following simple lemma and theorem present a deep insight into the structure of regular attack assignments.

Lemma 3 *If the attack relations assignments in $\mathcal{A}$ are regular then $\sqcup\mathcal{A}$ is also regular.*

Proof (Sketch) The proof is not difficult though rather lengthy as we just need to check in a straightforward way that each regular property is satisfied. $\Box$

It follows immediately

Theorem 2 *Suppose the set $RAT_{\mathcal{R}}$ of regular attack relation assignments is not empty. Then $(RAA_{\mathcal{R}}, \subseteq, \sqcup)$ is an upper semilattice.* $\Box$

Definition 18 *Suppose the set $RAA_{\mathcal{R}}$ of all regular attack relation assignments for $\mathcal{R}$ is not empty. The* **canonical attack relation assignment** *of $\mathcal{R}$ denoted by $\mathbf{Att}_{\mathcal{R}}$ is defined by:* $\mathbf{Att}_{\mathcal{R}} = \sqcup RAA_{\mathcal{R}}$.

Even though in general, regular attack relation assignments (and hence the canonical one) may not exist (as the example 7 below shows), they exist under natural conditions that we believe most practical rule-based systems satisfy, like the property of self-contradiction or closure under transposition or contraposition (see theorem 3 below).

Example 7 *Consider a rule-based system $\mathcal{R}$ consisting of $d_0 :\Rightarrow a \quad d_1 :\Rightarrow b$ $r : a \rightarrow \neg b$ and $d_0 \prec d_1$. Suppose atts be a regular attack relation assignment for $\mathcal{C}_{\mathcal{R}}$. Let $K = (\mathcal{R}, \emptyset)$. The arguments for K are given in figure 5. From the property of effective rebut, it is clear that $(A, B) \notin att(K)$. Hence $att(K) = \emptyset$. The inconsistency-resolving property is not satisfied by att, contradicting the assumption that atts is regular. Therefore there exists no regular attack relation assignment for $\mathcal{C}_K$.*

$$\begin{array}{ll} A: \neg b & B: b \\ \quad \uparrow_r & \quad \Uparrow_{d_1} \\ \quad a & \\ \quad \Uparrow_{d_0} & \end{array}$$

Figure 5. Non-existence of regular assignments

It turns out that a special type of attack relations, the normal attack relations introduced in [12] is regular if the rule-based systems is closed under transposition or contraposition or self-contradiction.

Let K be a knowledge base and $A, B \in AR_K$. We say that A *normal-rebuts* B (at X) iff A rebuts B (at X) and there is no defeasible rule $d \in ldr(A)$ such that $d \prec last(X)$.

The *normal attack relation assignment* [12] $atts_{nr}$ is defined by: For any knowledge base $K \in \mathcal{R}$ and any arguments $A, B \in AR_K$, $(A, B) \in atts_{nr}(K)$ if and only if A undercuts B or A normal-rebuts B.

We present below a central result of this paper.

Theorem 3 *Suppose the rule-based system $\mathcal{R}$ satisfies the self-contradiction property. Then the normal attack relation assignment $atts_{nr}$ is regular and the canonical assignment $Att_{\mathcal{R}}$ exists and $atts_{nr} \subseteq Att_{\mathcal{R}}$.*

Proof (Sketch) From theorem 2 and the definition of the canonical attack relation, we only need to show that $atts_{nr}$ is regular.

It is straightforward to show that for each $K \in \mathcal{C}_{\mathcal{R}}$, the attack relation $atts_{nr}(K)$ satisfies the properties of strong subargument structure, attack monotonicity, effective rebuts and link-orientation. Further it is also obvious that $atts_{nr}$ satisfies the context-independence property. Let $K \in \mathcal{C}_{\mathcal{R}}$. We show that $atts_{nr}(K)$ satisfies the inconsistency-resolving property. Let $S \subseteq AR_K$ s.t. S is inconsistent. Let S' be the set of all basic defeasible subarguments of S and S_0 be a minimal inconsistent subset of S'. Let $A \in S_0$ s.t. last(A) is minimal (wrt $\prec$) in $\{last(X) \mid X \in S_0\}$. From the self-contradiction property, $cnl(S_0) \vdash \neg hd(last(A))$.

We could then construct an argument B such that B attacks A and all basic defeasible subarguments of B are subarguments of arguments in S_0. $\square$.

Though the normal and canonical attack relations do not coincide in general, they are equivalent in the sense that they have identical sets of stable extensions.

Theorem 4 *Suppose the rule-based system $\mathcal{R}$ satisfies the property of self-contradiction. Then for each $K \in \mathcal{C}_{\mathcal{R}}$, $E \subseteq AR_K$ is a stable extension wrt $atts_{nr}(K)$ iff E is a stable extension wrt $Att_{\mathcal{R}}(K)$.*

Proof (Sketch) We first show that for each $atts \in RAA_{\mathcal{R}}$, each stable extension of $(AR_K, atts(K))$ is also a stable extension of $(AR_K, atts_{nr}(K))$. Hence each stable extension of $(AR_K, Att_{\mathcal{R}}(K))$ is also stable extension of $(AR_K, atts_{nr}(K))$. The theorem follows then from lemma 4 below. $\square$

Lemma 4 *Let $atts, atts'$ be regular attack relation assignments for $\mathcal{R}$ such that $atts \subseteq atts'$. Then* (1.) *each stable extension of $(AR_K, atts(K))$ is a stable extension of $(AR_K, atts'(K))$; and* (2.) *each stable extension of $(AR_K, atts(K))$ is a stable extension of $(AR_K, Att_{\mathcal{R}}(K))$.*

Proof (Sketch) 1) Let E be a stable extension of $(AR_K, atts(K))$. It is clear that E attacks each argument in $AR_K \setminus E$ wrt $atts'(K)$. If E is not conflict-free wrt $atts'(K)$, E is inconsistent (since both $atts, atts'$ have the same set of undercuts) and hence not conflict-free wrt atts(K) (a contradiction). Hence E is conflict-free (and hence stable) wrt $atts'(K)$. 2) Follows immediately from (1) and the definition of $Att_{\mathcal{R}}$. $\square$

6. Discussion and Conclusion

The preference-based approaches to argumentation [2,3,24,22,23] define the semantics of defeasible knowledge bases by first defining a preference relation between arguments and then using the preference relation to define attack relation between arguments. We could also define an argument preference assignment for a rule-based system $\mathcal{R}$ as a function assigning to each knowledge base $K \in \mathcal{C}_{\mathcal{R}}$, a relation $\sqsubseteq_K \subseteq AR_K \times AR_K$ representing a preference relation between arguments in AR_K where strict arguments are not strictly less preferred than any other arguments. It is possible to define a lower semilattice over the set of preference relation assignments whose least element corresponds to the canonical semantics (see [13]).

A key property satisfied by many argument-based and non-argument-based approaches to reasoning with prioritized rules is the credulous cumulativity property [12] stating intuitively that if some beliefs in your belief set are confirmed in the reality then your belief set will not change because of it. We show in [13] that credulous cumulativity is satisfied by regular attack relation assignments.

A more liberal notion of rebut, referred to as unrestricted rebut, where a basic defeasible argument could directly attack a non-basic defeasible argument is studied in [9,8]. Intuitively an unrestricted rebut is a rebut against a set of defeasible rules without explicitly rebutting any individual rule in it. It would be interesting to see how this notion of rebut interacts with the regular properties.

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